SCIENCE
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Exploring Large Integer Multiplication for Cryptography Targeting In-Memory Computing

Florian Krieger, Florian Hirner, Sujoy Sinha Roy
Institute of Information Security, TU Graz

Outline

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FHE and ZKP: Promising but Challenging

- FHE and ZKP offer novel opportunities
- But there are limitations:
  - Huge computational overhead ($10^5\times$ or more) [1]
  - Lots of data involved ($\sim$ 10GB) [2]

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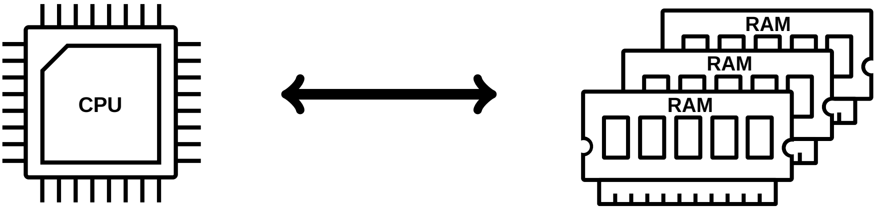

- Substantial data streams
- 100× energy overhead [3]
- Latency bottleneck

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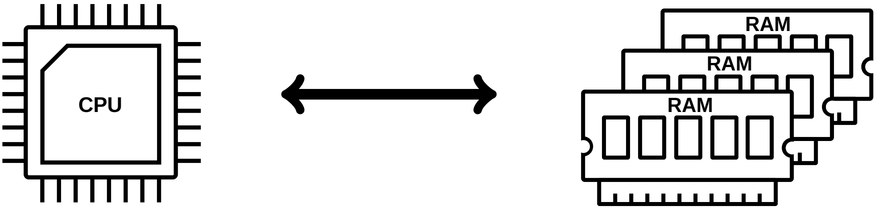

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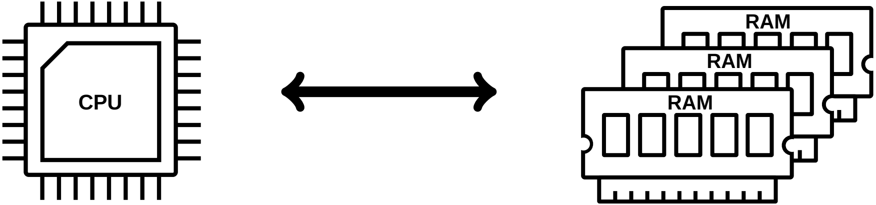

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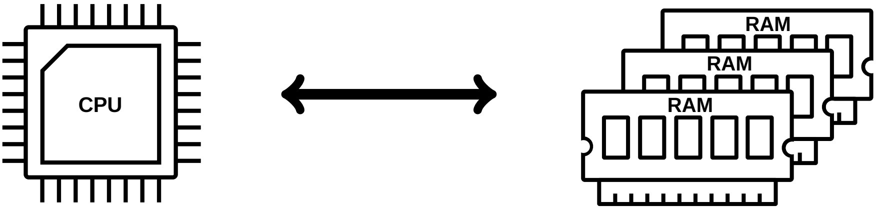

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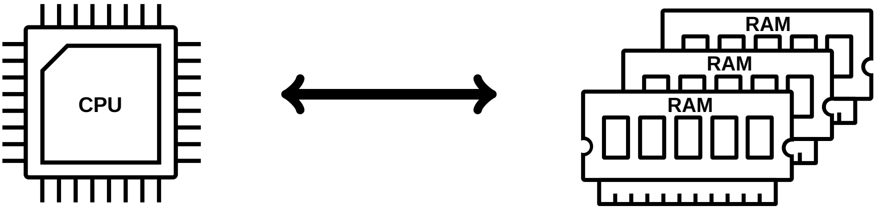

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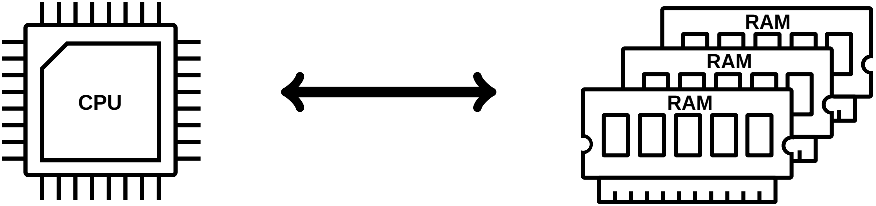

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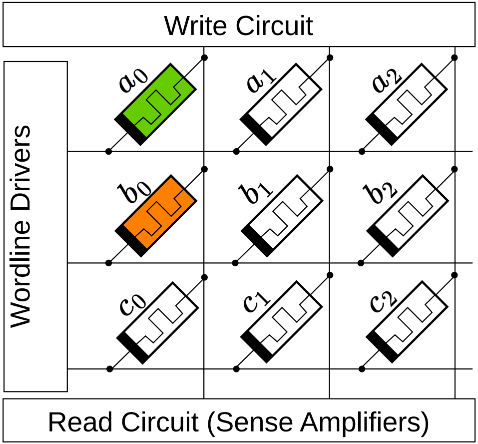
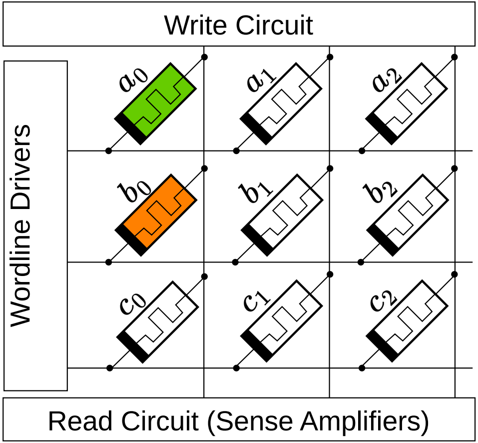
In-Memory Computing!

Resistive Memory: Basic Functionality



Logic 0 $\Leftrightarrow$ HR
Logic 1 $\Leftrightarrow$ LR

Resistive Memory: Basic Functionality [4]



a	b	$c = \mathrm{NOR}(a, b)$
0 (HR)	0 (HR)	1 (LR)
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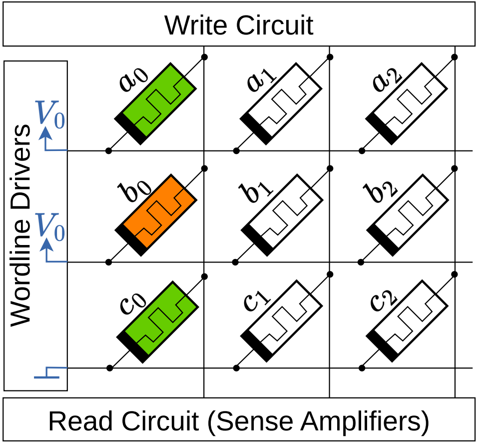

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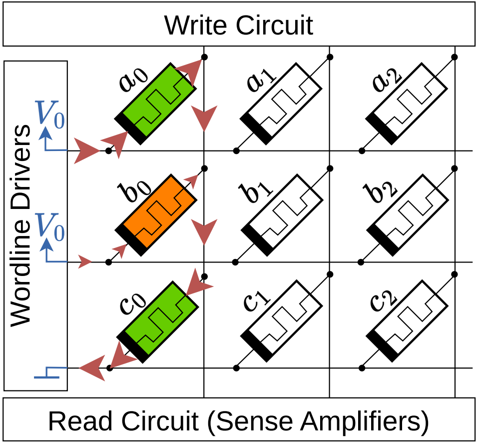

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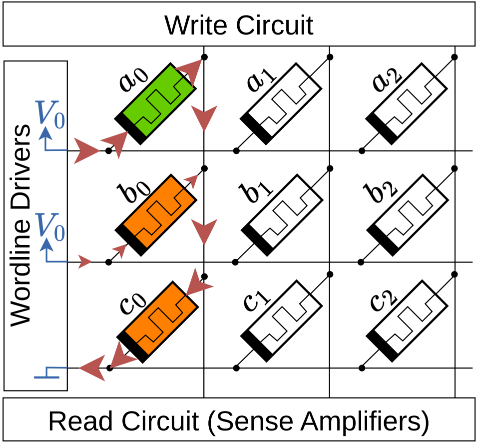

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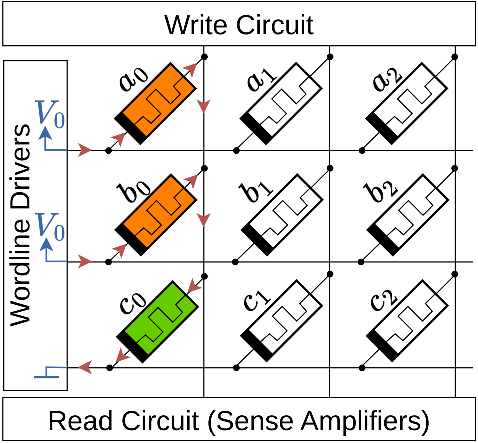

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In-Memory Computing for FHE and ZKP

Challenges:

- High-level arithmetic from low-level logic gates
  - ➔ Hundreds of bits per operand
- No FHE/ZKP friendly multiplier presented
  - ➔ Typically relying on schoolbook multiplication: $O(n^2)$
- Endurance of cells
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Goal: Close research gap for large integer multipliers

Contributions:

- Algorithmic exploration
- Large integer multiplier design for IMC
  - Algorithmic optimizations
  - Implementation-specific optimizations

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- Split operands into k chunks
- Evaluate $\rightarrow$ Multiply $\rightarrow$ Interpolate

- ⊕ Good asymptotic complexity
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e.g. $k = 3$:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/2 & 1/3 & -1 & 1/6 & -2 \\ -1 & 1/2 & 1/2 & 0 & -1 \\ -1/2 & 1/6 & 1/2 & -1/6 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Algorithmic Exploration: Karatsuba Multiplication

- Special case of Toom-Cook: $k = 2$
- Slightly slower than Toom-Cook
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Selected Karatsuba

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Our Kogge-Stone Adder

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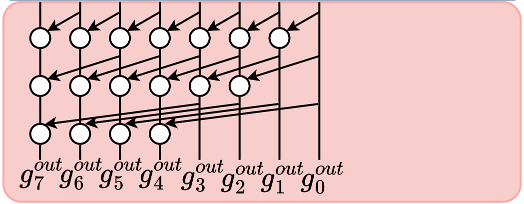

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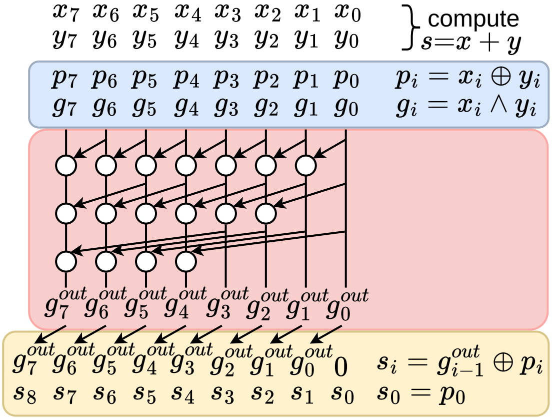

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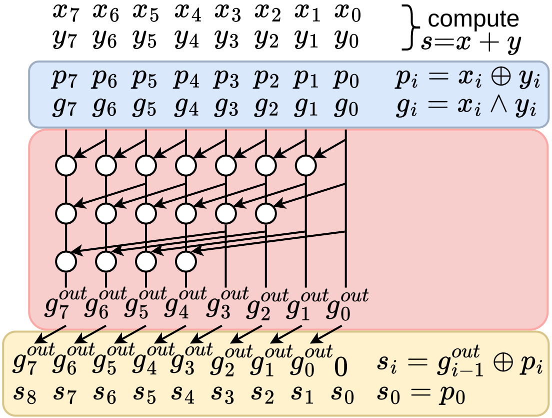
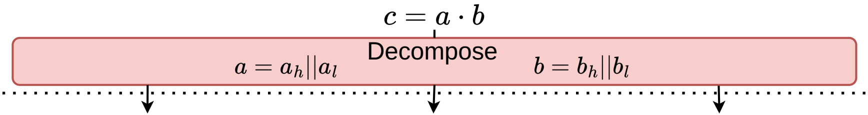


- Logarithmic depth
- Regular structure
- Good choice for IMC

Karatsuba Multiplier Design for IMC



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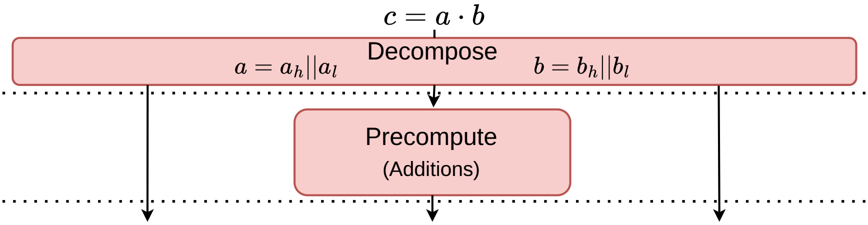

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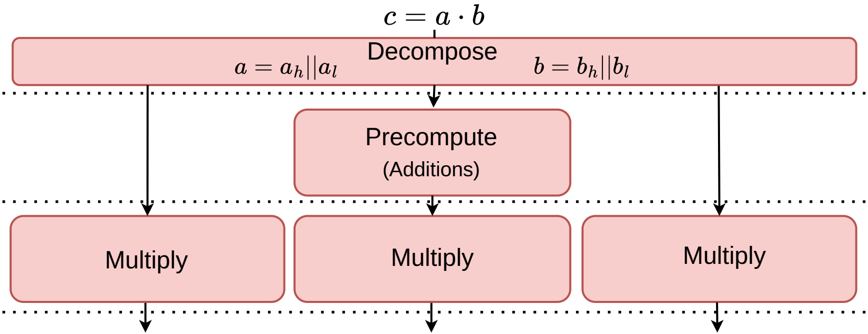

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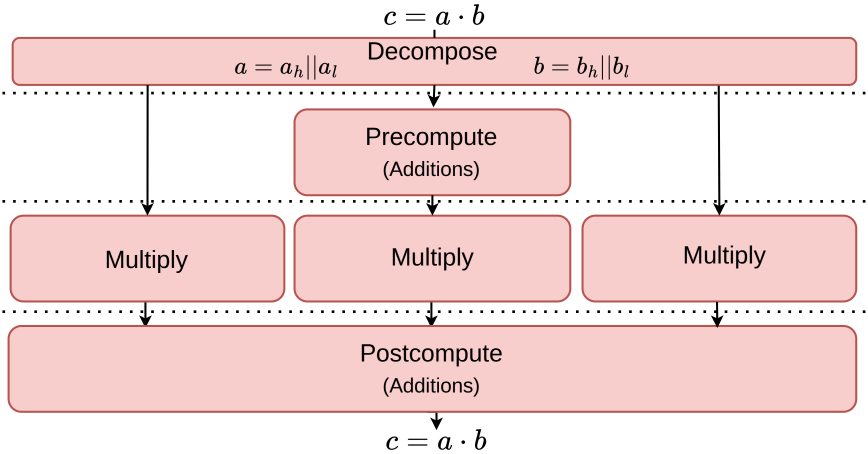

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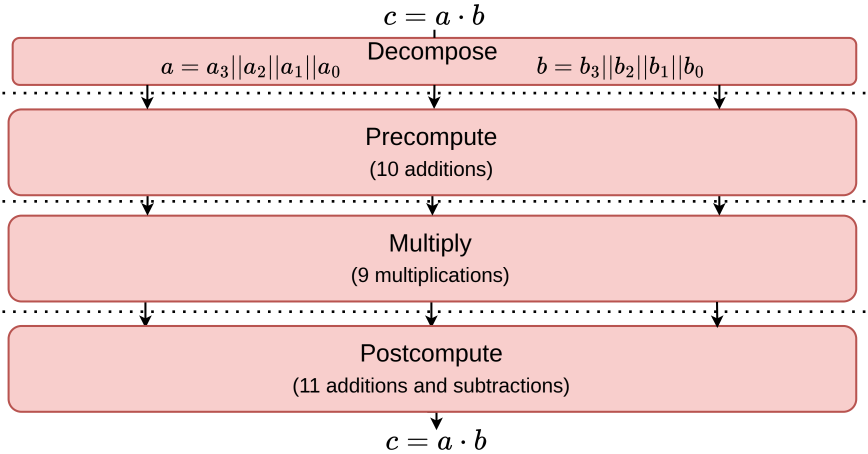

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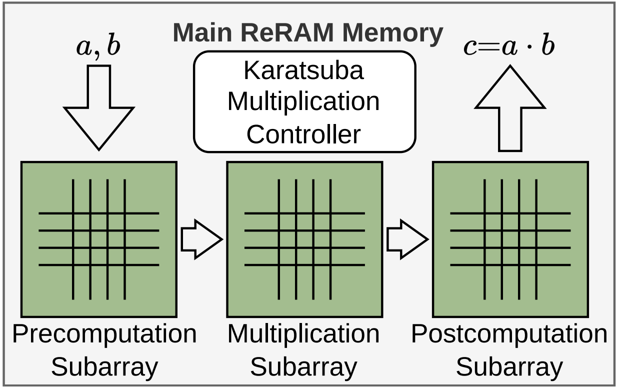

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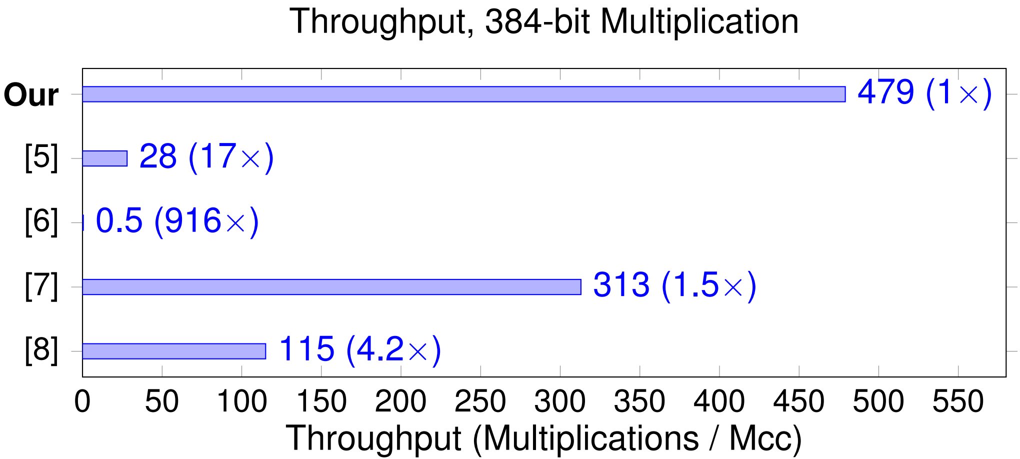
Comparison with Related Work



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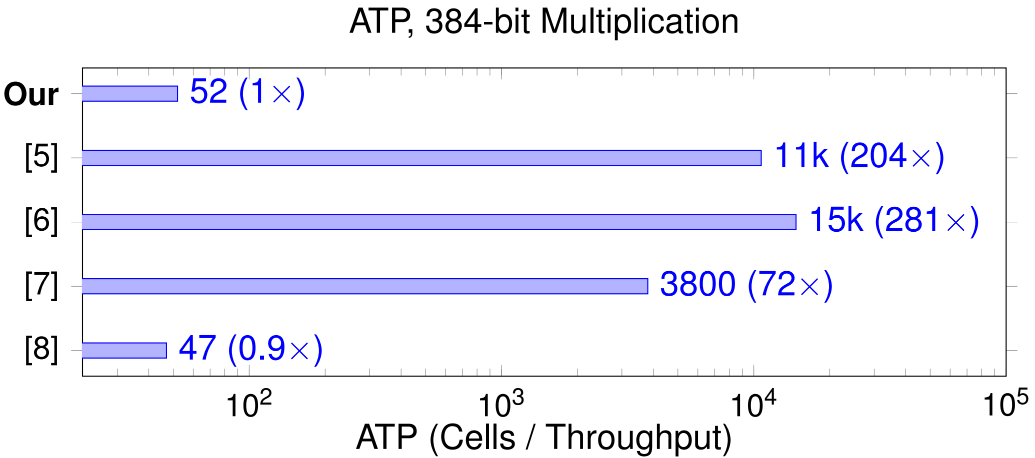

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- Building block for modular arithmetic
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References

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