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SCIENCE
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TECHNOLOGY

Accelerating Hash-Based Polynomial Commitment Schemes with Linear Prover Time

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Graz University of Technology

CHES 2025 Conference

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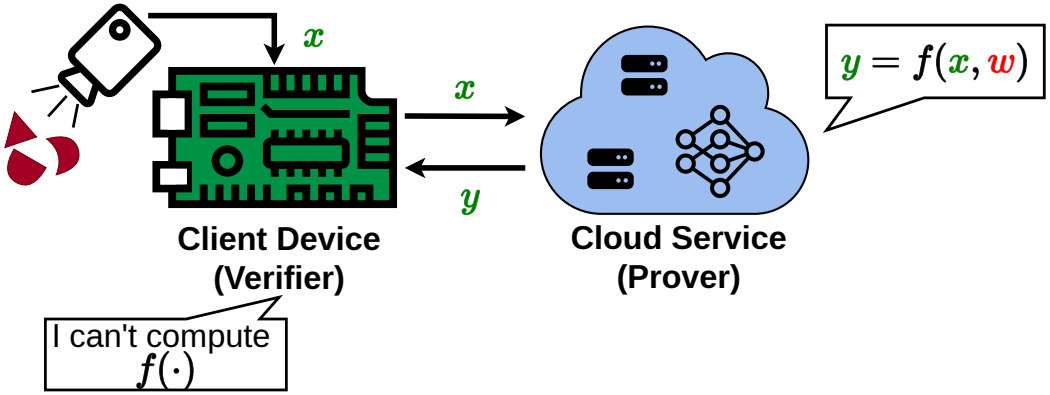
Verifiable Cloud Computing



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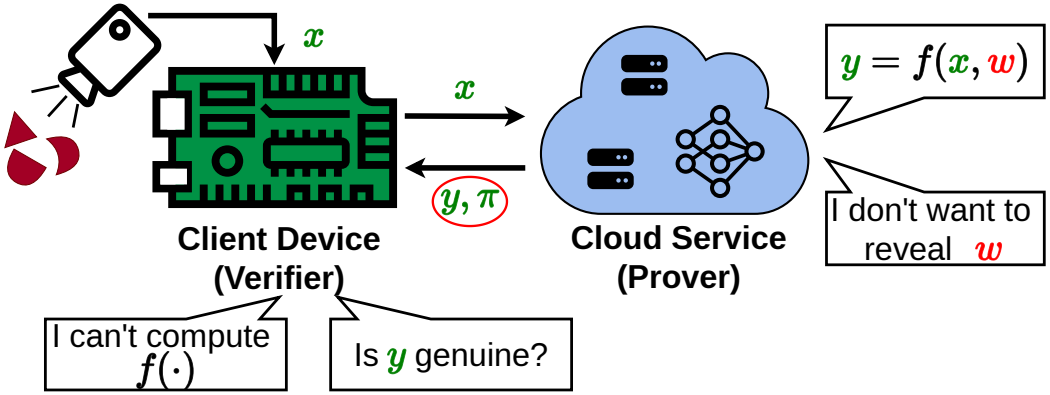

Polynomial Commitment Schemes

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→**Cause substantial latency**

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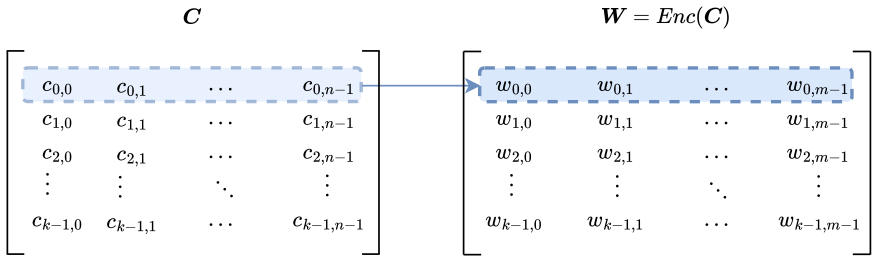

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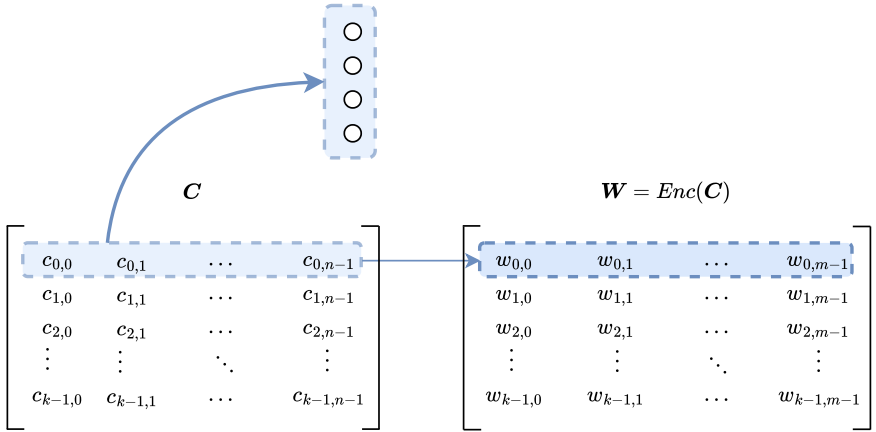

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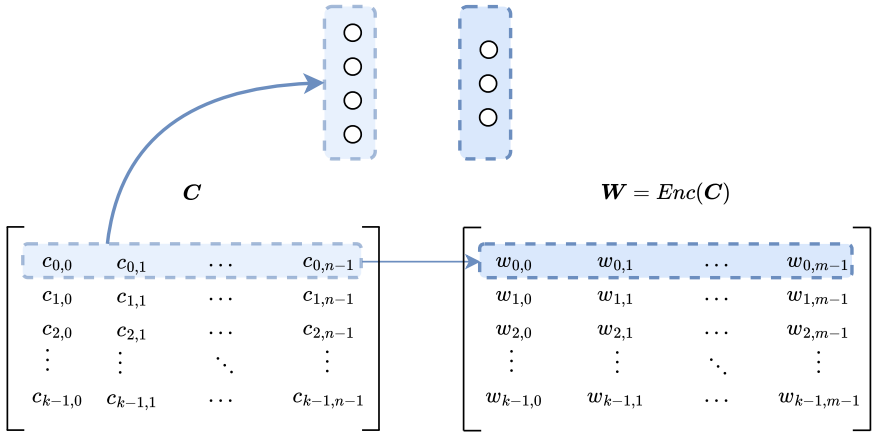

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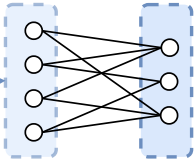

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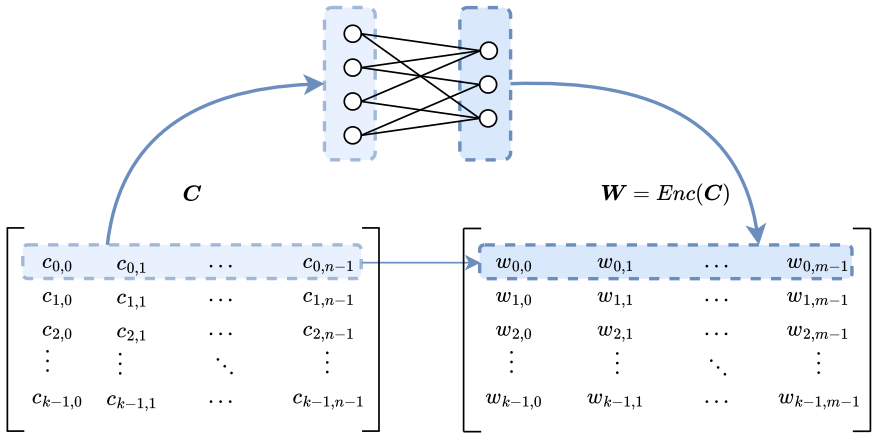

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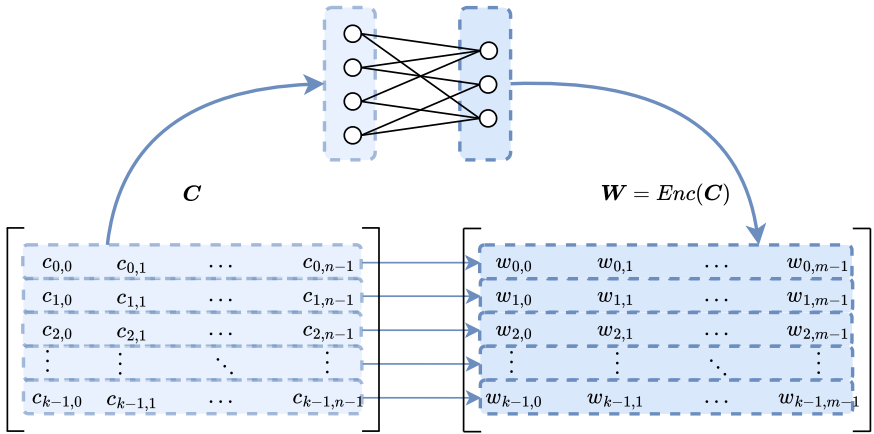

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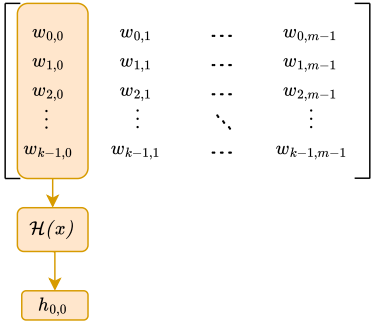

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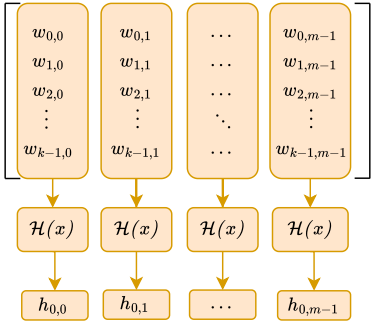

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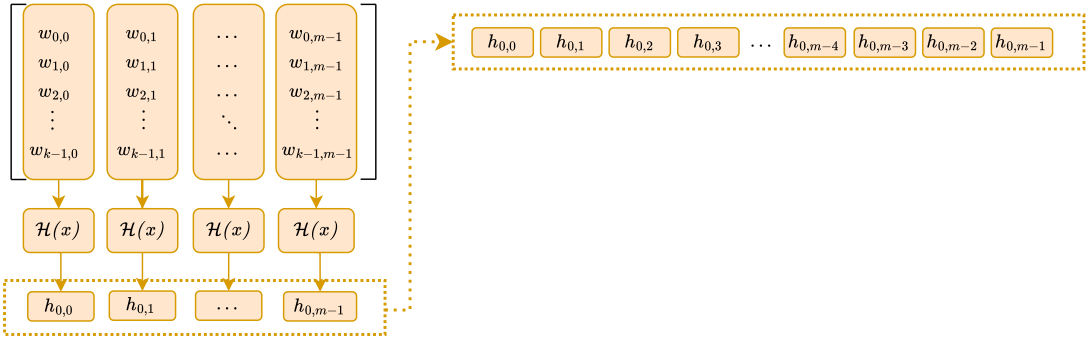

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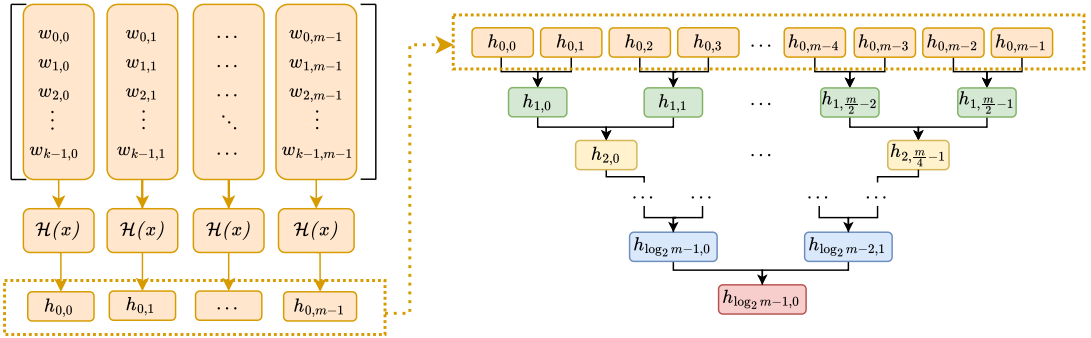

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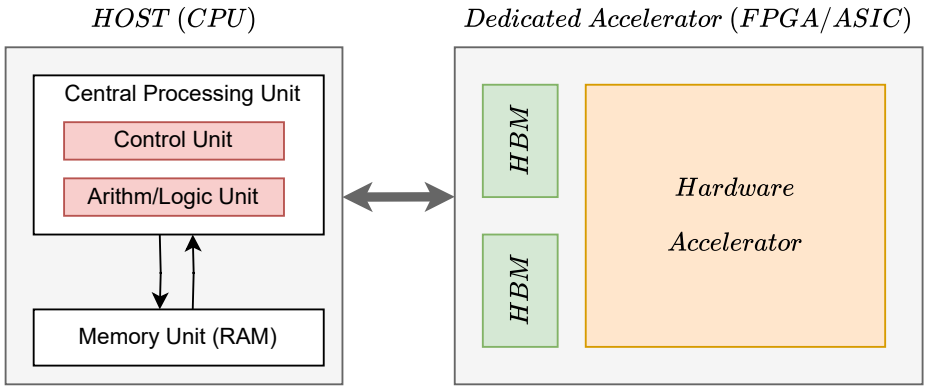
We address these limiting factors

→ Memory- and Platform-Aware Hardware Design!

Computational Data Flow of PCS



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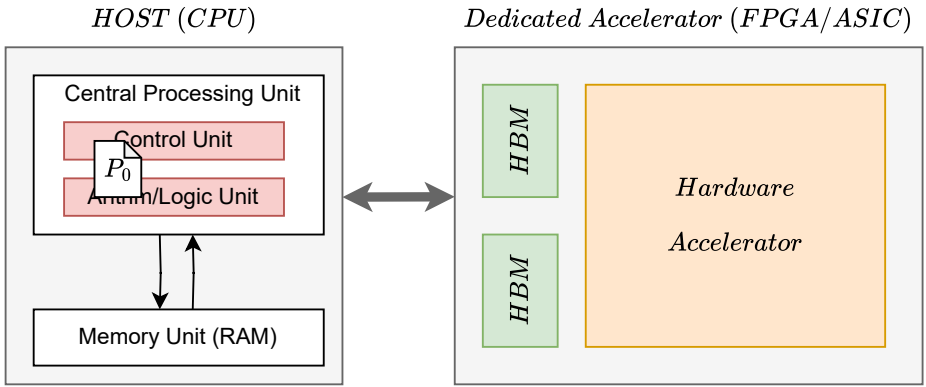

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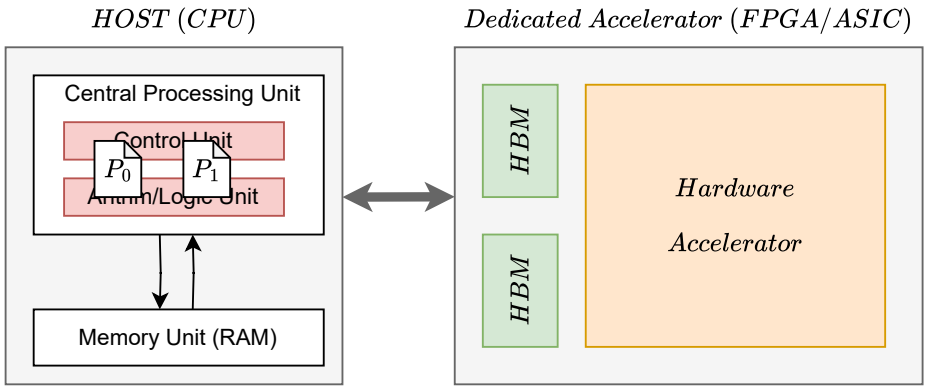

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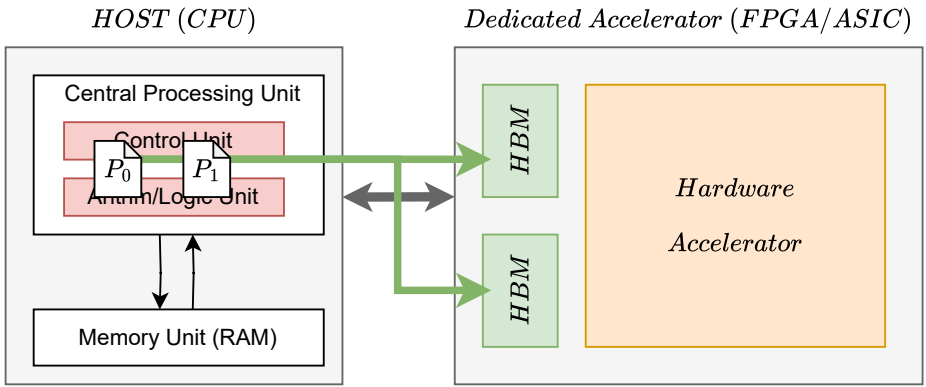

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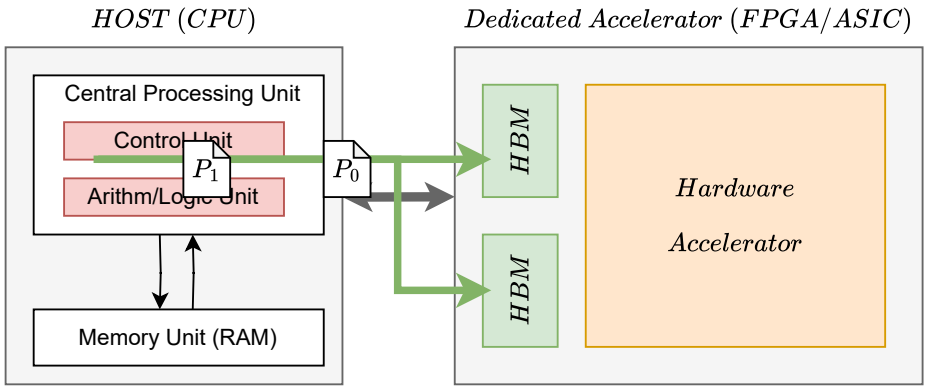

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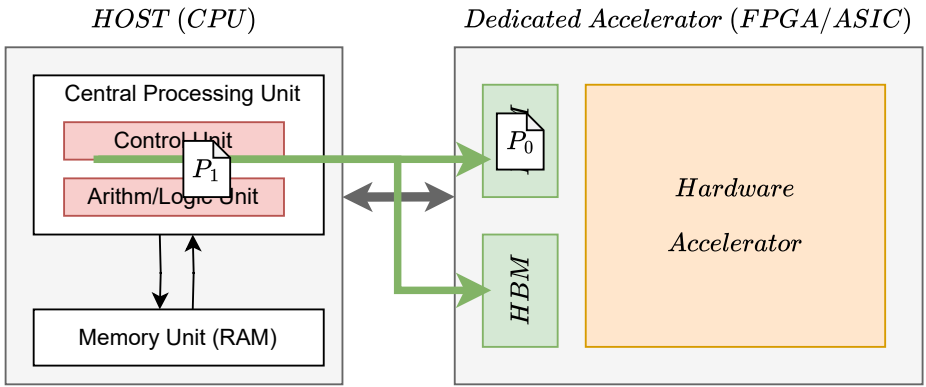

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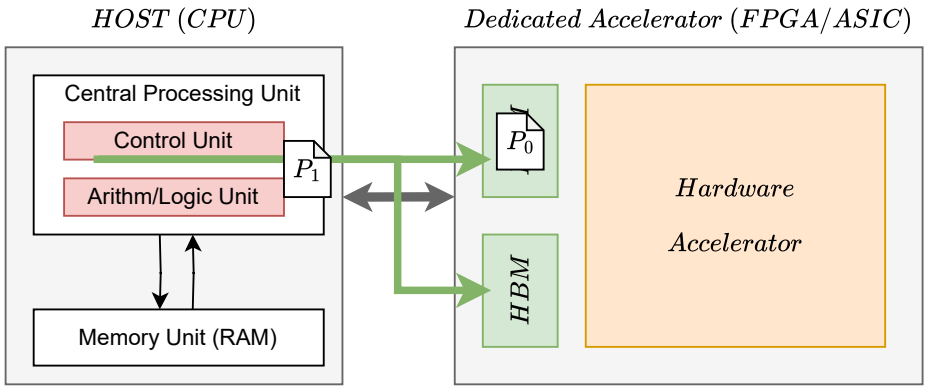

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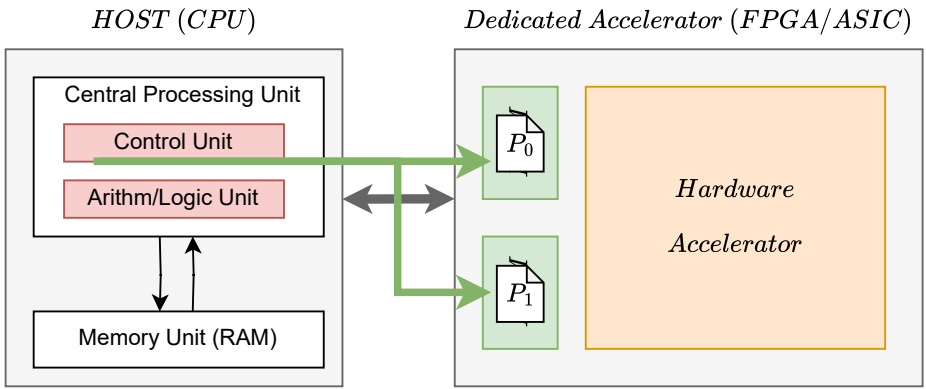

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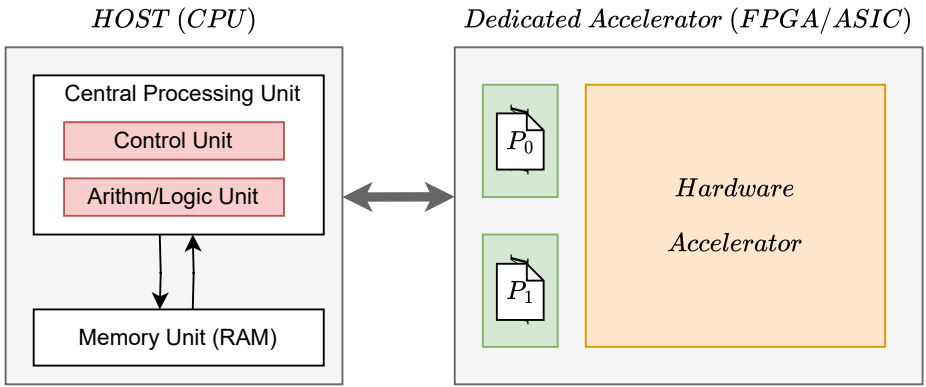

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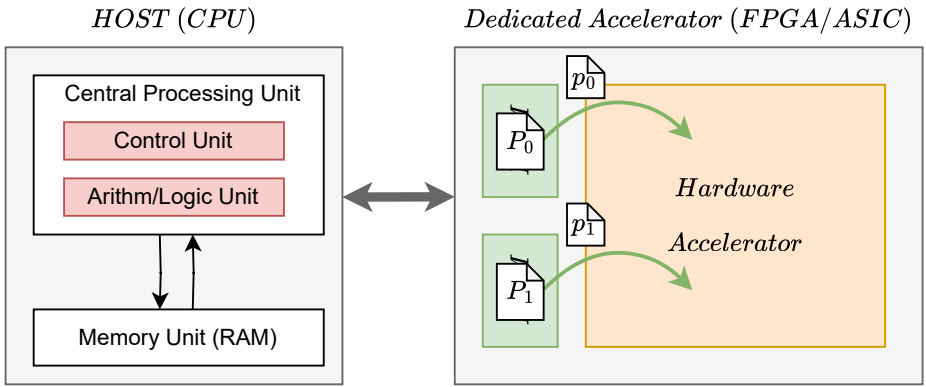

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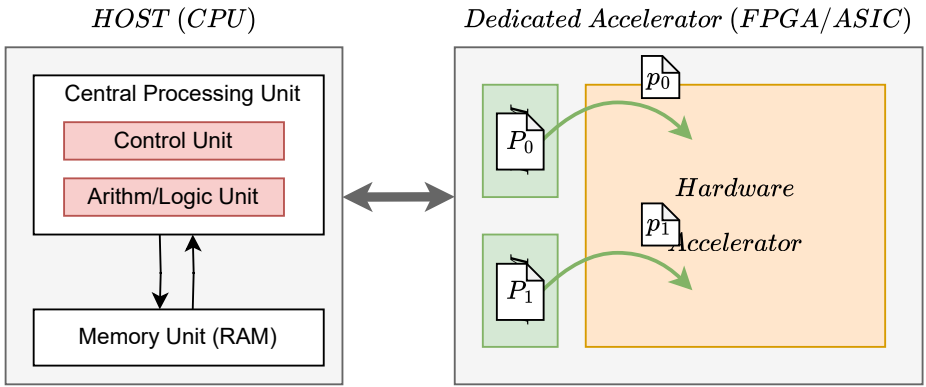

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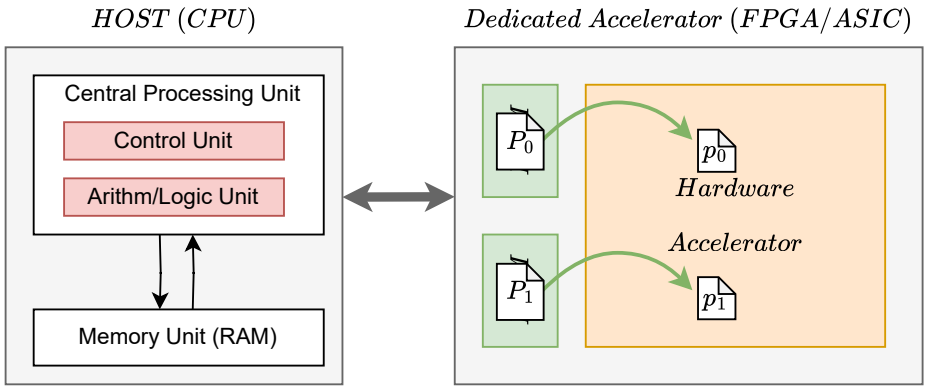

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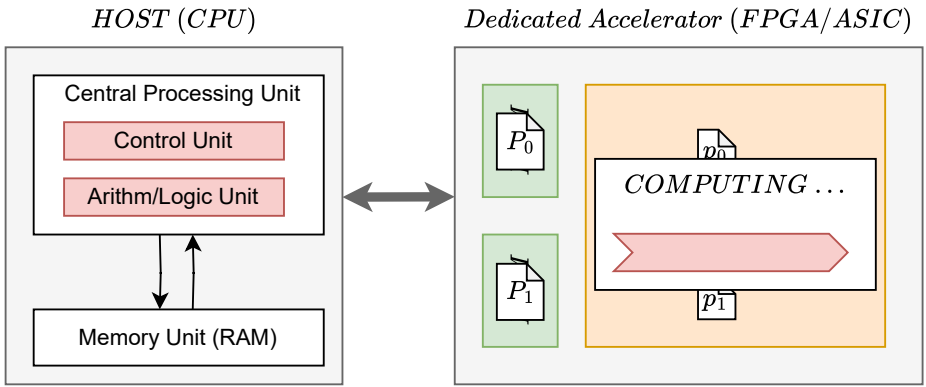

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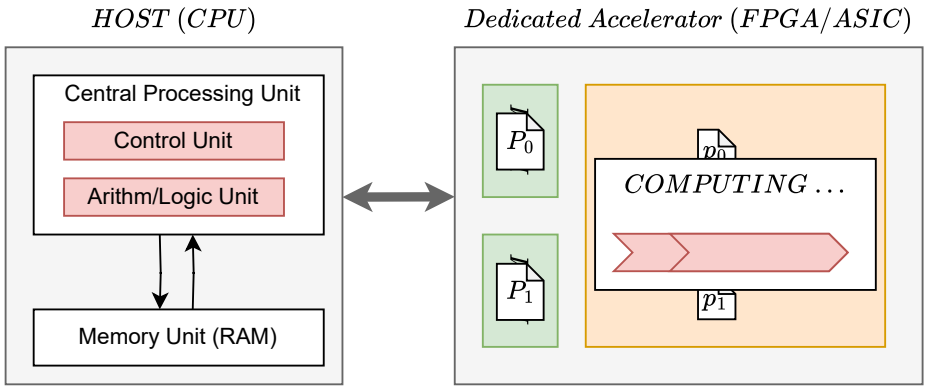

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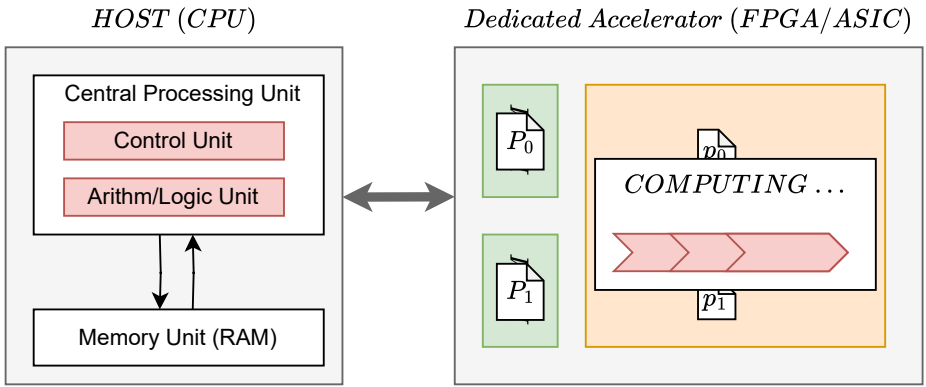

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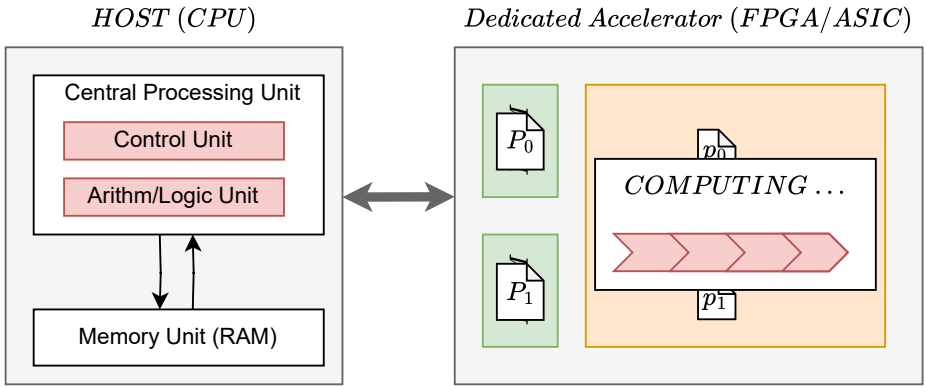

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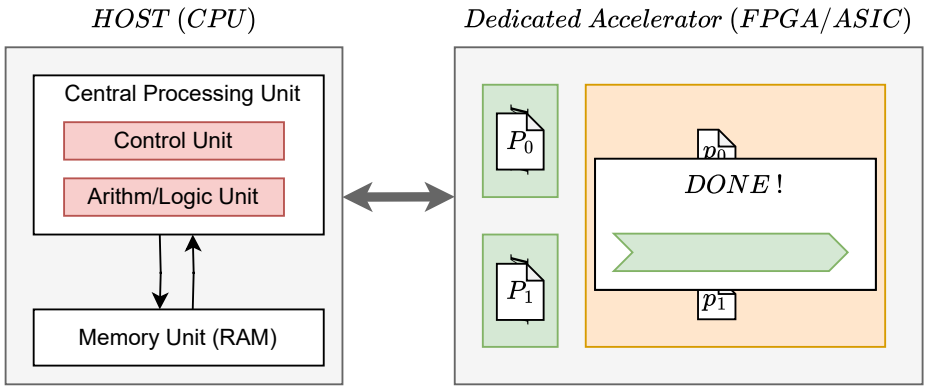

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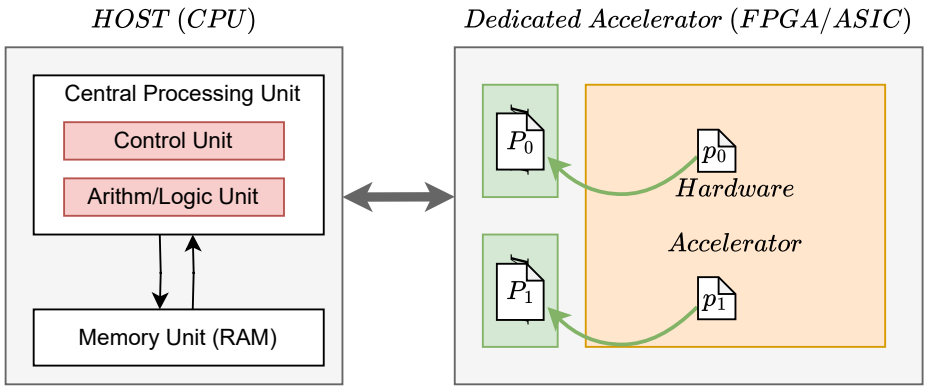

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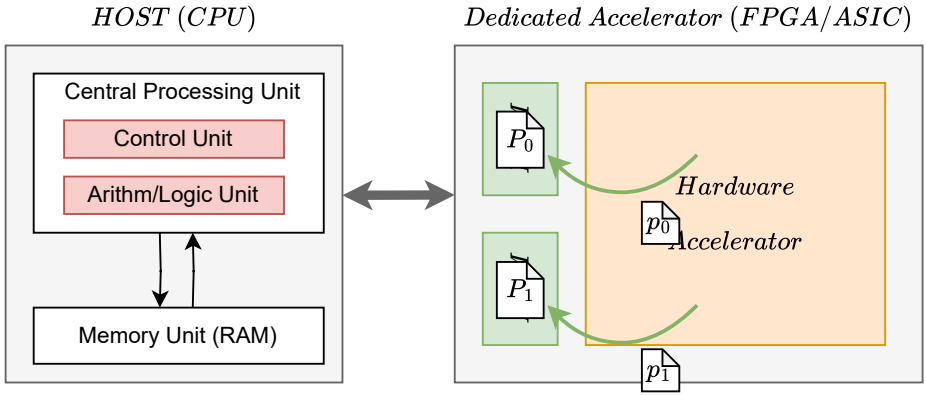

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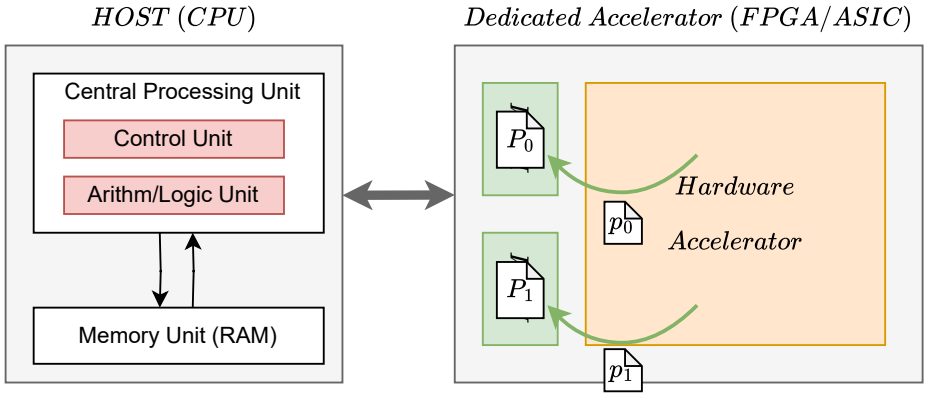

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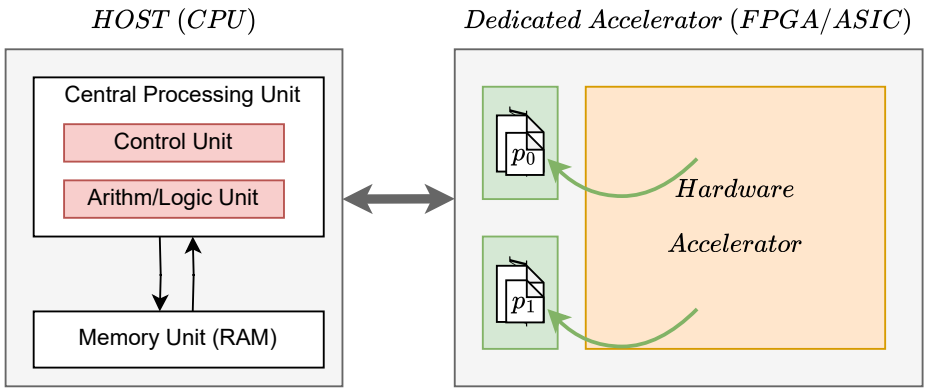

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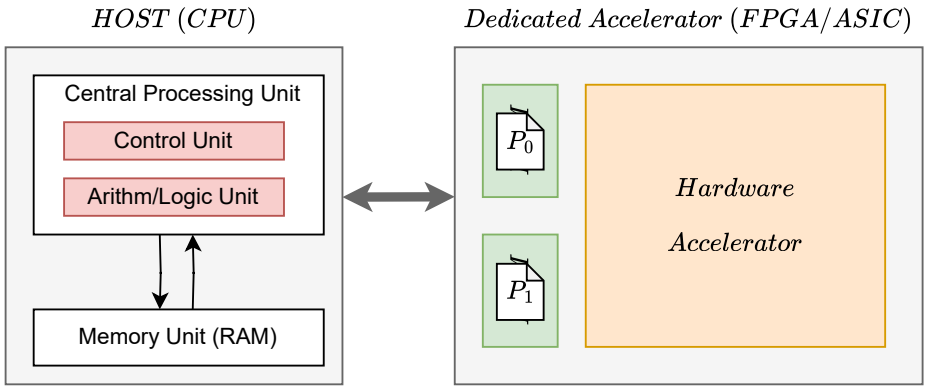

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Achievable bandwidth (rule-of-thumb)

Access pattern	**% of peak BW**
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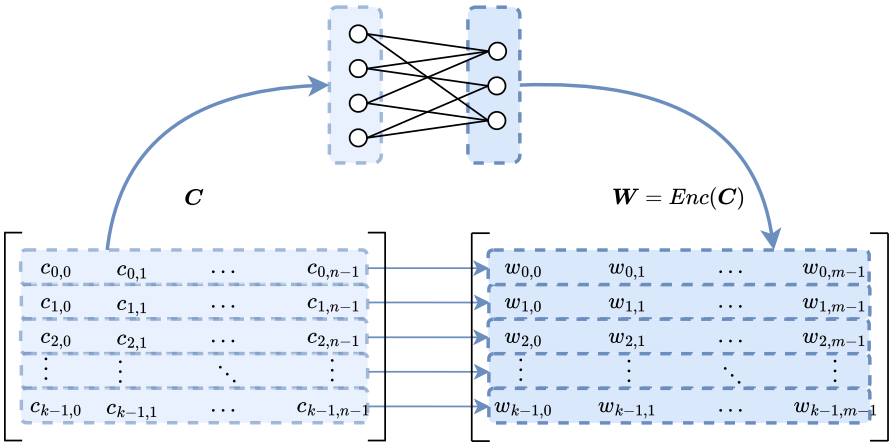
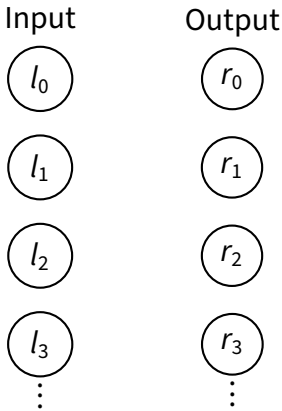
Linear Encoding heavily uses random access

Linear Encoding using Expander Graphs



Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



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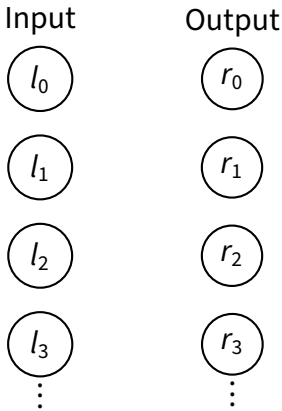

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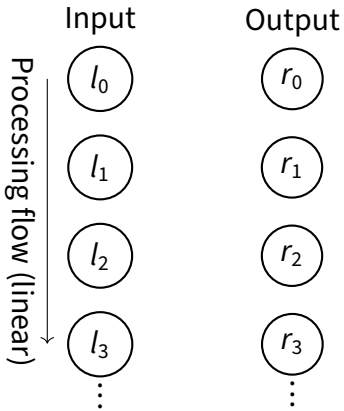
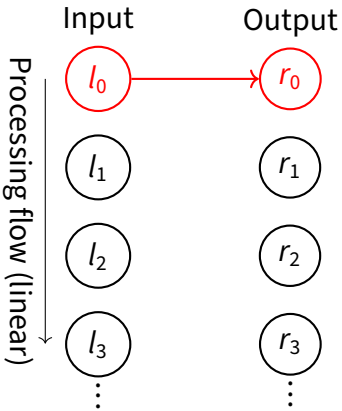


Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

$$r_i = \sum \omega_{i,j} \cdot l_j$$

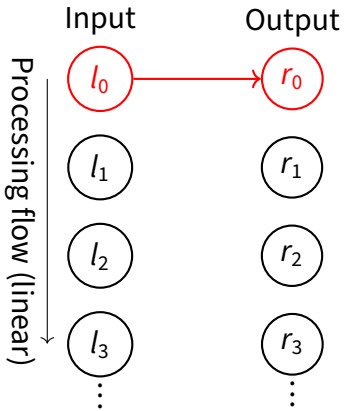


Step 1: Rd(l_0) |

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

$$r_i = \sum \omega_{i,j} \cdot l_j$$

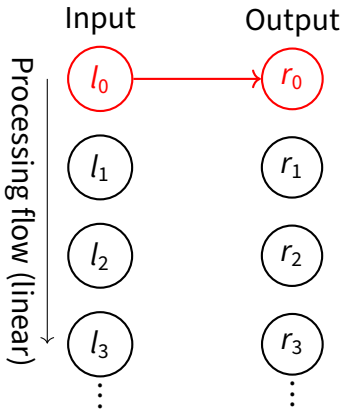


Step 1: Rd(l_0) | Rd(r_0) |

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

$$r_i = \sum \omega_{i,j} \cdot l_j$$

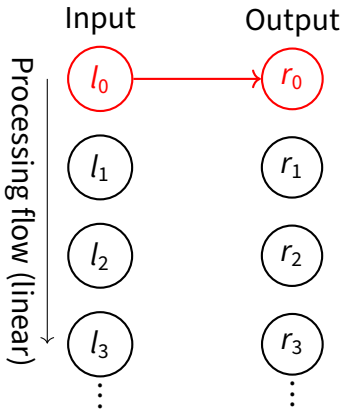


Step 1: Rd(l_0) | Rd(r_0) | r_0+= $\omega_{0,0} l_0$ |

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

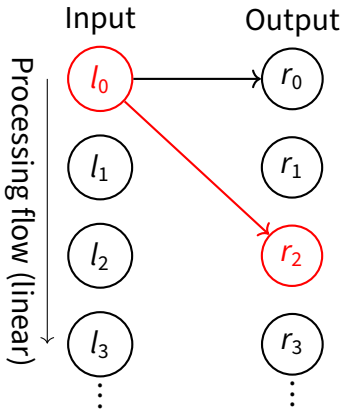
$$r_i = \sum \omega_{i,j} \cdot l_j$$



Step 1: $\mathrm{Rd}(l_0) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \mathrm{Wr}(r_0)$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

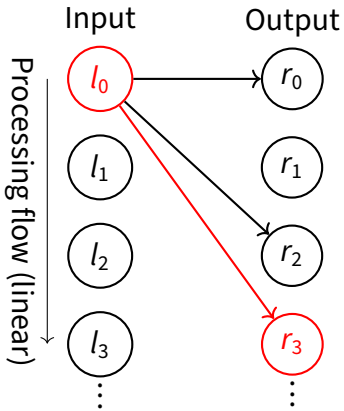
Step 1: $\mathrm{Rd}(l_0) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \mathrm{Wr}(r_0)$

Step 2: $\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \mathrm{Wr}(r_2)$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

$$r_i = \sum \omega_{i,j} \cdot l_j$$



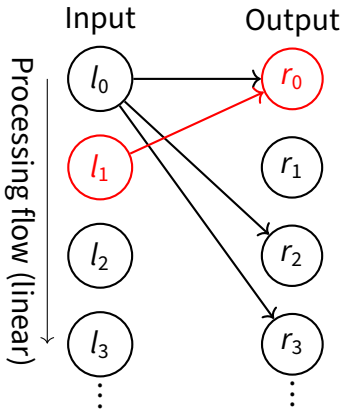
Step 1: $\text{Rd}(l_0) \mid \text{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \text{Wr}(r_0)$

Step 2: $\text{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \text{Wr}(r_2)$

Step 3: $\text{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,0} l_0 \mid \text{Wr}(r_3)$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\text{Rd}(l_0) \mid \text{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \text{Wr}(r_0)$

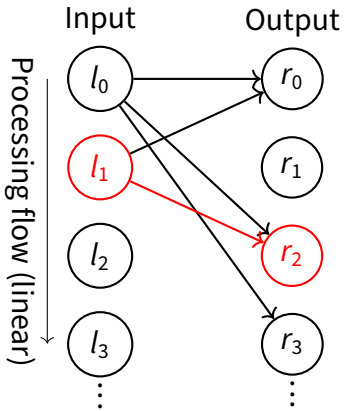
Step 2: $\text{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \text{Wr}(r_2)$

Step 3: $\text{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,0} l_0 \mid \text{Wr}(r_3)$

Step 4: $\text{Rd}(l_1) \mid \text{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,1} l_1 \mid \text{Wr}(r_0)$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\mathrm{Rd}(l_0) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \mathrm{Wr}(r_0)$

Step 2: $\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \mathrm{Wr}(r_2)$

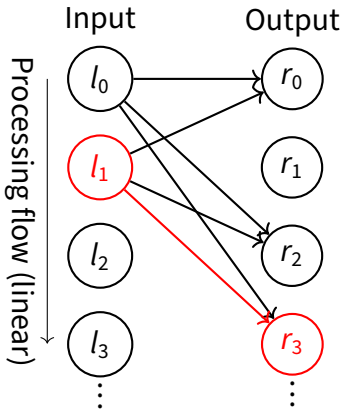
Step 3: $\mathrm{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,0} l_0 \mid \mathrm{Wr}(r_3)$

Step 4: $\mathrm{Rd}(l_1) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,1} l_1 \mid \mathrm{Wr}(r_0)$

Step 5: $\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,1} l_1 \mid \mathrm{Wr}(r_2)$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R

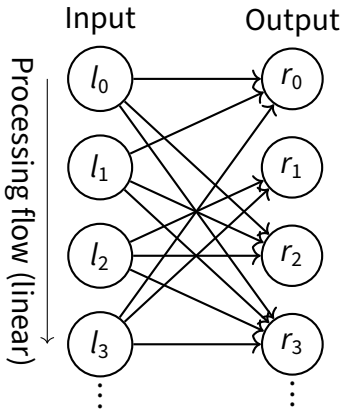


$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1:	Rd(l_0)	Rd(r_0)	$r_0 \mathrel{+}= \omega_{0,0} l_0$	Wr(r_0)
Step 2:		Rd(r_2)	$r_2 \mathrel{+}= \omega_{2,0} l_0$	Wr(r_2)
Step 3:		Rd(r_3)	$r_3 \mathrel{+}= \omega_{3,0} l_0$	Wr(r_3)
Step 4:	Rd(l_1)	Rd(r_0)	$r_0 \mathrel{+}= \omega_{0,1} l_1$	Wr(r_0)
Step 5:		Rd(r_2)	$r_2 \mathrel{+}= \omega_{2,1} l_1$	Wr(r_2)
Step 6:		Rd(r_3)	$r_3 \mathrel{+}= \omega_{3,1} l_1$	Wr(r_3)

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\mathrm{Rd}(l_0) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \mathrm{Wr}(r_0)$
Step 2: $\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \mathrm{Wr}(r_2)$
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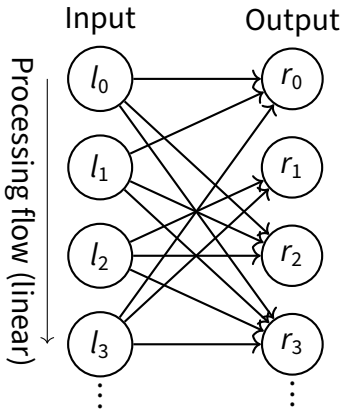
Step 4: $\mathrm{Rd}(l_1) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,1} l_1 \mid \mathrm{Wr}(r_0)$
Step 5: $\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,1} l_1 \mid \mathrm{Wr}(r_2)$
Step 6: $\mathrm{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,1} l_1 \mid \mathrm{Wr}(r_3)$
$\ldots$

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

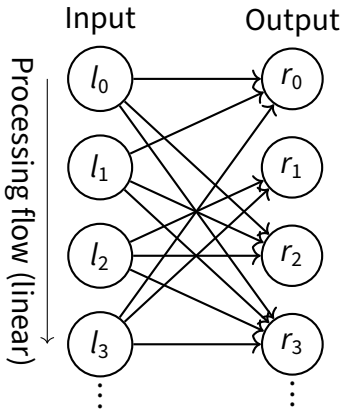
Step 1:	$\mathrm{Rd}(l_0) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0 \mid \mathrm{Wr}(r_0)$
Step 2:	$\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,0} l_0 \mid \mathrm{Wr}(r_2)$
Step 3:	$\mathrm{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,0} l_0 \mid \mathrm{Wr}(r_3)$
Step 4:	$\mathrm{Rd}(l_1) \mid \mathrm{Rd}(r_0) \mid r_0 \mathrel{+}= \omega_{0,1} l_1 \mid \mathrm{Wr}(r_0)$
Step 5:	$\mathrm{Rd}(r_2) \mid r_2 \mathrel{+}= \omega_{2,1} l_1 \mid \mathrm{Wr}(r_2)$
Step 6:	$\mathrm{Rd}(r_3) \mid r_3 \mathrel{+}= \omega_{3,1} l_1 \mid \mathrm{Wr}(r_3)$
	...

Problems of L2R:

- ⛔ Many random Rd / Wr

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

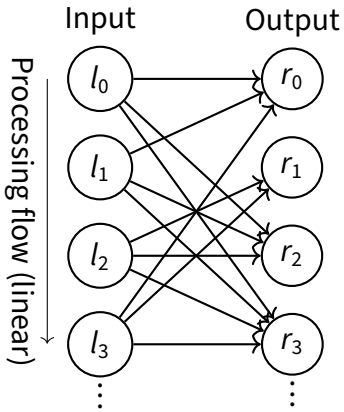
Step 1:	$\mathrm{Rd}(l_0)$	$\mathrm{Rd}(r_0)$	$r_0 \mathrel{+}= \omega_{0,0} l_0$ | $\mathrm{Wr}(r_0)$
Step 2:		$\mathrm{Rd}(r_2)$	$r_2 \mathrel{+}= \omega_{2,0} l_0$ | $\mathrm{Wr}(r_2)$
Step 3:		$\mathrm{Rd}(r_3)$	$r_3 \mathrel{+}= \omega_{3,0} l_0$ | $\mathrm{Wr}(r_3)$
Step 4:	$\mathrm{Rd}(l_1)$	$\mathrm{Rd}(r_0)$	$r_0 \mathrel{+}= \omega_{0,1} l_1$ | $\mathrm{Wr}(r_0)$
Step 5:		$\mathrm{Rd}(r_2)$	$r_2 \mathrel{+}= \omega_{2,1} l_1$ | $\mathrm{Wr}(r_2)$
Step 6:		$\mathrm{Rd}(r_3)$	$r_3 \mathrel{+}= \omega_{3,1} l_1$ | $\mathrm{Wr}(r_3)$
	...		

Problems of L2R:

- ⛔ Many random Rd / Wr
- ⛔ Rd / Wr turnarounds

Expander Graphs: The Core Operation in Spielman Codes

Baseline Graph: L2R



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1:	Rd(l_0)	Rd(r_0)	r_0+=$\omega_{0,0} l_0$	Wr(r_0)
Step 2:		Rd(r_2)	r_2+=$\omega_{2,0} l_0$	Wr(r_2)
Step 3:		Rd(r_3)	r_3+=$\omega_{3,0} l_0$	Wr(r_3)
Step 4:	Rd(l_1)	Rd(r_0)	r_0+=$\omega_{0,1} l_1$	Wr(r_0)
Step 5:		Rd(r_2)	r_2+=$\omega_{2,1} l_1$	Wr(r_2)
Step 6:		Rd(r_3)	r_3+=$\omega_{3,1} l_1$	Wr(r_3)
	...			

Problems of L2R:

- ⛔ Many random Rd / Wr
- ⛔ Rd / Wr turnarounds
- ⛔ Read-after-Write hazards

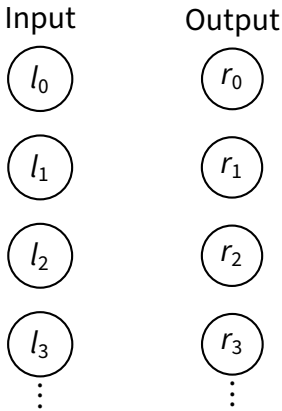
Our Inverted Expander Graphs

R2L Evaluation

Inverted Expander Graphs

Inverted Graph: R2L

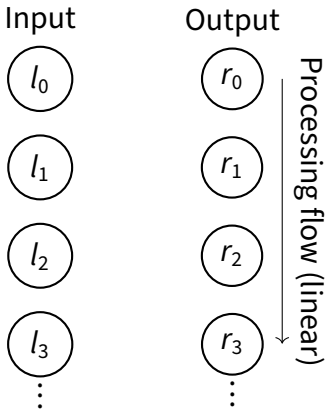
$$r_i = \sum \omega_{i,j} \cdot l_j$$



Inverted Expander Graphs

Inverted Graph: R2L

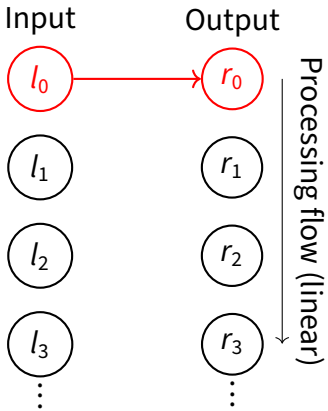
$$r_i = \sum \omega_{i,j} \cdot l_j$$



Inverted Expander Graphs

Inverted Graph: R2L

$$r_i = \sum \omega_{i,j} \cdot l_j$$

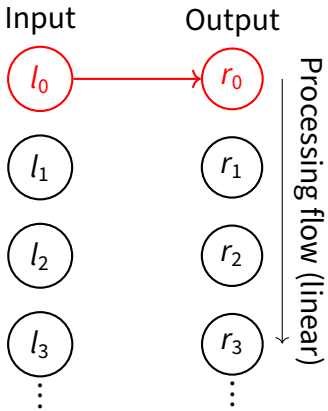


Step 1: $\mathrm{Rd}(l_0)$ |

Inverted Expander Graphs

Inverted Graph: R2L

$$r_i = \sum \omega_{i,j} \cdot l_j$$

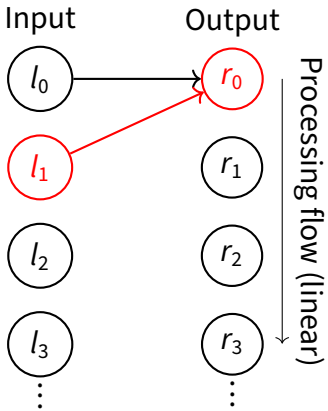


Step 1: $\text{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$

Inverted Expander Graphs

Inverted Graph: R2L

$$r_i = \sum \omega_{i,j} \cdot l_j$$

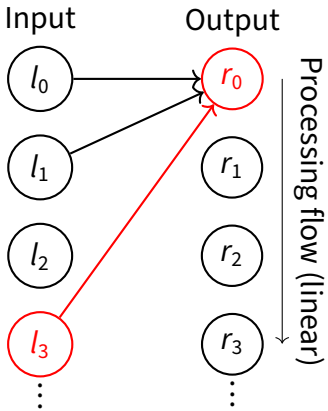


Step 1: $\text{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$

Step 2: $\text{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$

Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

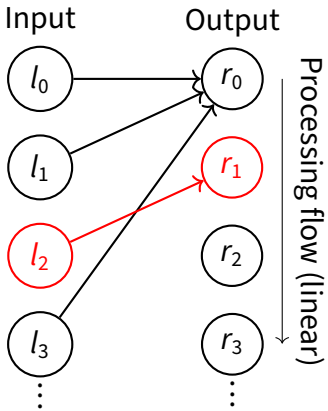
Step 1: $\text{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$

Step 2: $\text{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$

Step 3: $\text{Rd}(l_3) \mid r_0 \mathrel{+}= \omega_{0,3} l_3 \mid \text{Wr}(r_0)$

Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\mathrm{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$

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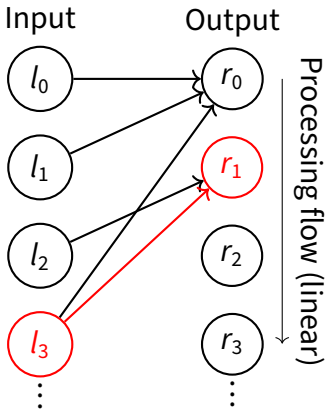
Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\text{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$
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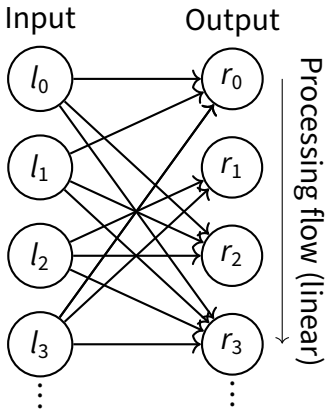
Step 4: $\text{Rd}(l_2) \mid r_1 \mathrel{+}= \omega_{1,2} l_2$
Step 5: $\text{Rd}(l_3) \mid r_1 \mathrel{+}= \omega_{1,3} l_3 \mid \text{Wr}(r_1)$

Inverted Expander Graphs

Inverted Graph: R2L

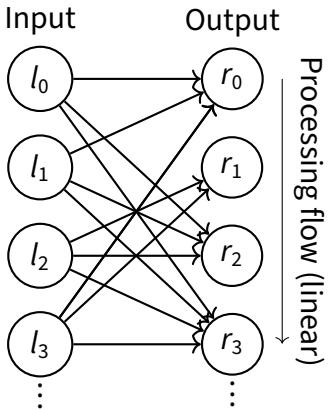


$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\mathrm{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$
Step 2: $\mathrm{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$
Step 3: $\mathrm{Rd}(l_3) \mid r_0 \mathrel{+}= \omega_{0,3} l_3 \mid \mathrm{Wr}(r_0)$
Step 4: $\mathrm{Rd}(l_2) \mid r_1 \mathrel{+}= \omega_{1,2} l_2$
Step 5: $\mathrm{Rd}(l_3) \mid r_1 \mathrel{+}= \omega_{1,3} l_3 \mid \mathrm{Wr}(r_1)$
$\ldots$

Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

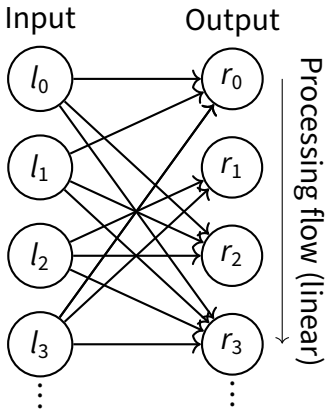
Step 1: $\mathrm{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$
Step 2: $\mathrm{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$
Step 3: $\mathrm{Rd}(l_3) \mid r_0 \mathrel{+}= \omega_{0,3} l_3 \mid \mathrm{Wr}(r_0)$
Step 4: $\mathrm{Rd}(l_2) \mid r_1 \mathrel{+}= \omega_{1,2} l_2$
Step 5: $\mathrm{Rd}(l_3) \mid r_1 \mathrel{+}= \omega_{1,3} l_3 \mid \mathrm{Wr}(r_1)$
$\ldots$

Benefits of R2L:

- ⊕ Only linear Wr

Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

Step 1: $\mathrm{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$
Step 2: $\mathrm{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$
Step 3: $\mathrm{Rd}(l_3) \mid r_0 \mathrel{+}= \omega_{0,3} l_3 \mid \mathrm{Wr}(r_0)$
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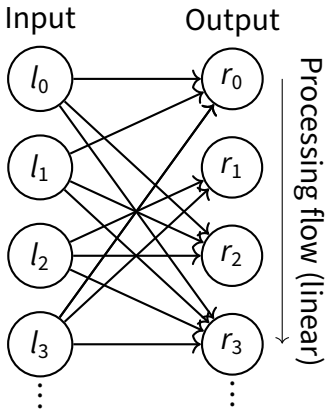
Step 5: $\mathrm{Rd}(l_3) \mid r_1 \mathrel{+}= \omega_{1,3} l_3 \mid \mathrm{Wr}(r_1)$
$\ldots$

Benefits of R2L:

- ⊕ Only linear Wr
- ⊕ Fewer turnarounds

Inverted Expander Graphs

Inverted Graph: R2L



$$r_i = \sum \omega_{i,j} \cdot l_j$$

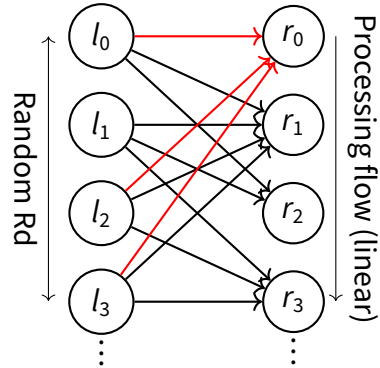
Step 1: $\text{Rd}(l_0) \mid r_0 \mathrel{+}= \omega_{0,0} l_0$
Step 2: $\text{Rd}(l_1) \mid r_0 \mathrel{+}= \omega_{0,1} l_1$
Step 3: $\text{Rd}(l_3) \mid r_0 \mathrel{+}= \omega_{0,3} l_3 \mid \text{Wr}(r_0)$
Step 4: $\text{Rd}(l_2) \mid r_1 \mathrel{+}= \omega_{1,2} l_2$
Step 5: $\text{Rd}(l_3) \mid r_1 \mathrel{+}= \omega_{1,3} l_3 \mid \text{Wr}(r_1)$
$\ldots$

Benefits of R2L:

- ⊕ Only linear Wr
- ⊕ Fewer turnarounds
- ⊕ No hazards

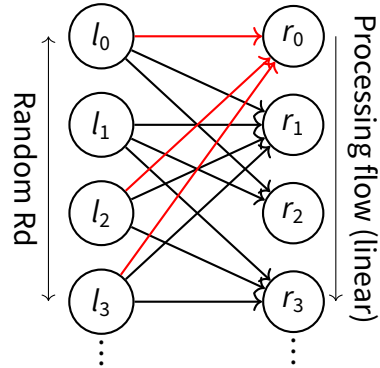
Inverted Expander Graphs: R2L

- R2L reduces off-chip memory accesses



Inverted Expander Graphs: R2L

- R2L reduces off-chip memory accesses
- How to handle graph structure?
  - Randomly generated for L2R



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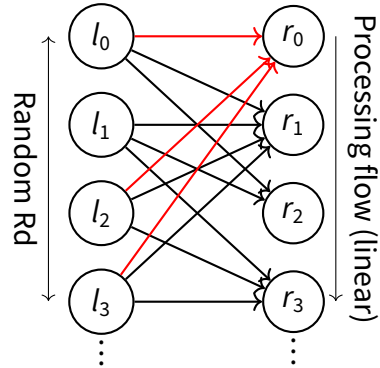

Inverted Expander Graphs: R2L

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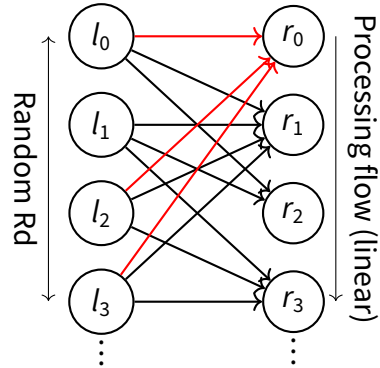
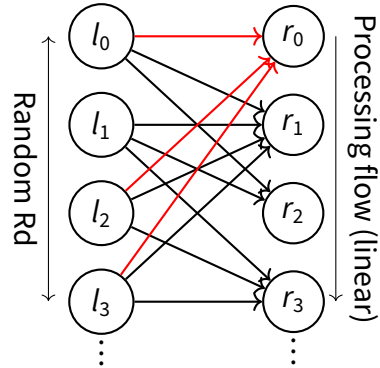
  1. R2L with stored inverted graph



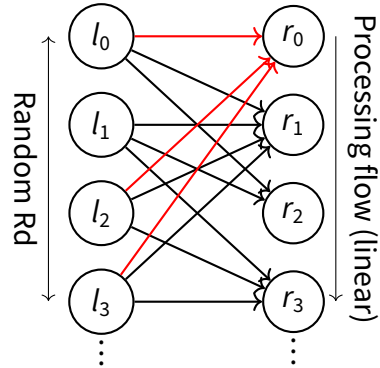
Inverted Expander Graphs: R2L

- R2L reduces off-chip memory accesses
- How to handle graph structure?
  - Randomly generated for L2R

1. R2L with stored inverted graph
2. R2L with generated graph + Postprocessing

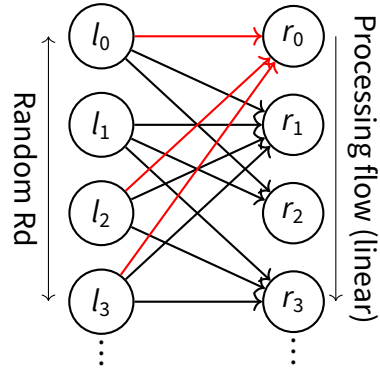


R2L: Stored Inverted Graph



R2L: Stored Inverted Graph

1. Randomly generated in **left-node** major order $(l_0, r_0), (l_0, r_1), (l_0, r_2), (l_1, r_1), \ldots$



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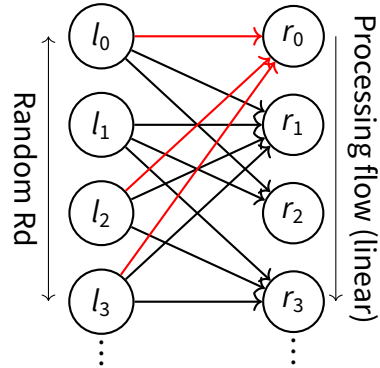

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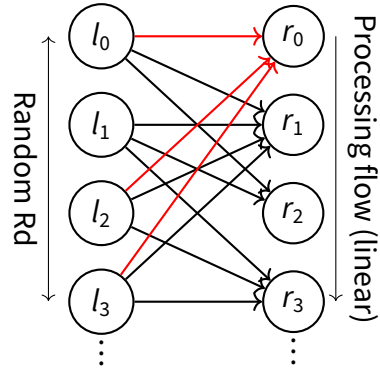
3. Store transformed graph



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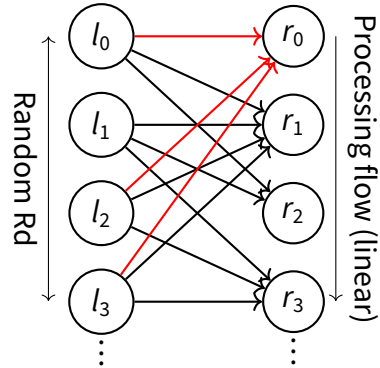
⊕ Identical result



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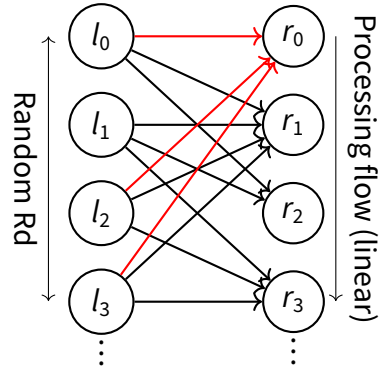
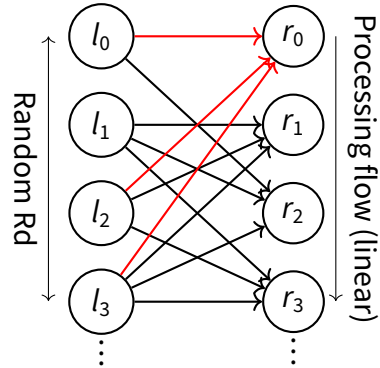
⊕ Identical result

⊖ Must be stored in off-chip memory (Gigabytes!)

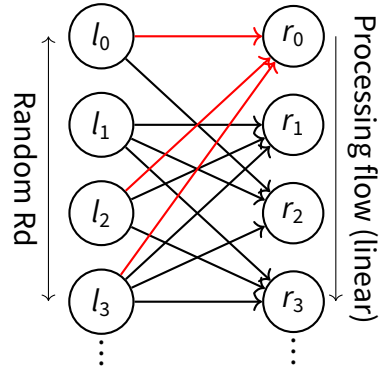


R2L: Generated Graph + Postprocessing



R2L: Generated Graph + Postprocessing

1. Fix right node degree



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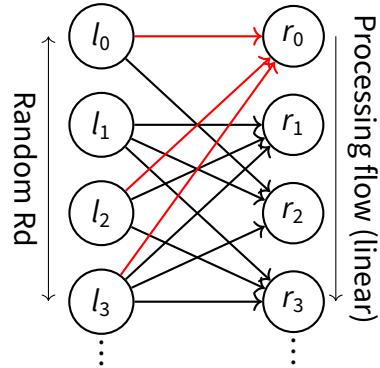

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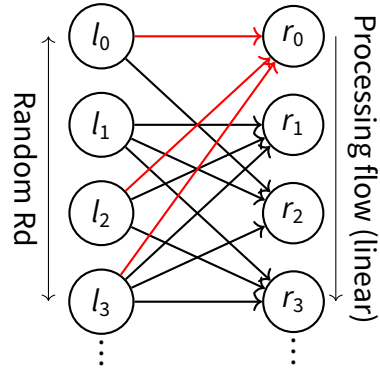
⊕ Generated using small PRNG



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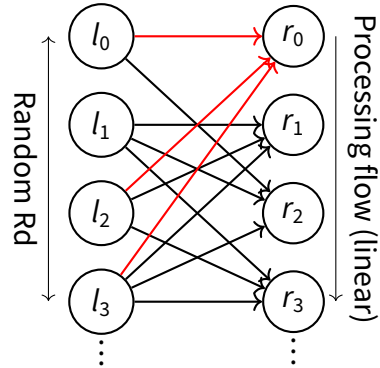
- ⊕ Generated using small PRNG
- ⊕ No off-chip graph memory!



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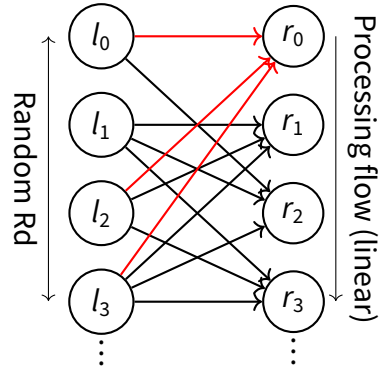
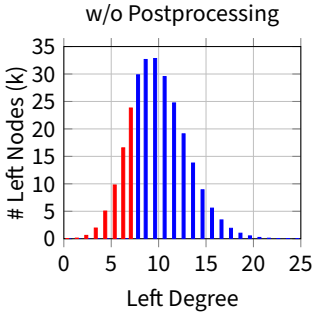
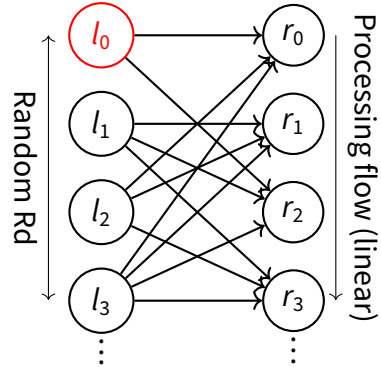
- ? Security



R2L: Generated Graph + Postprocessing

- Some l_i have low degree





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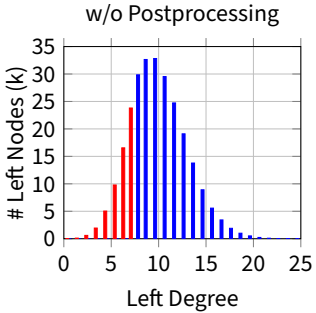
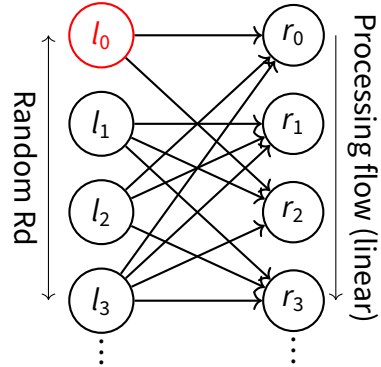
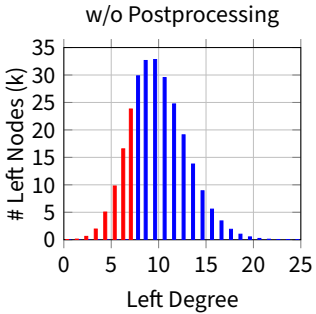
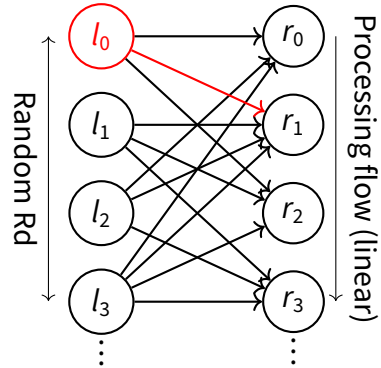




R2L: Generated Graph + Postprocessing

- Some l_i have low degree
- Add connections via "Postprocessing"
  - Store l_i in on-chip memory





R2L: Generated Graph + Postprocessing

- Some l_i have low degree
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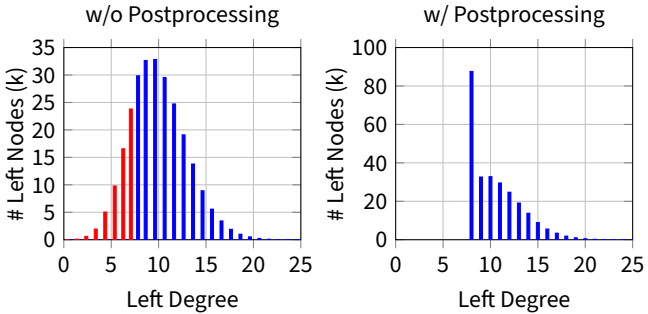
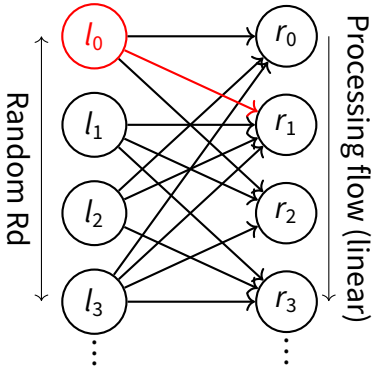
- Each l_i has **at least** a certain degree





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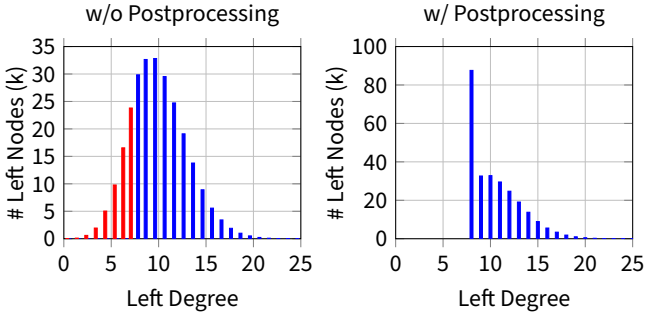
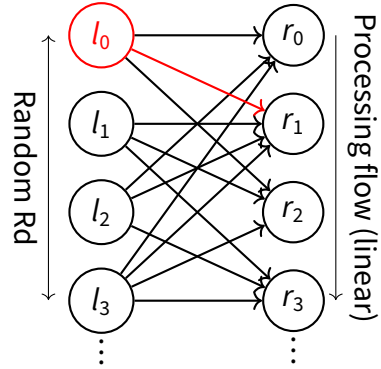
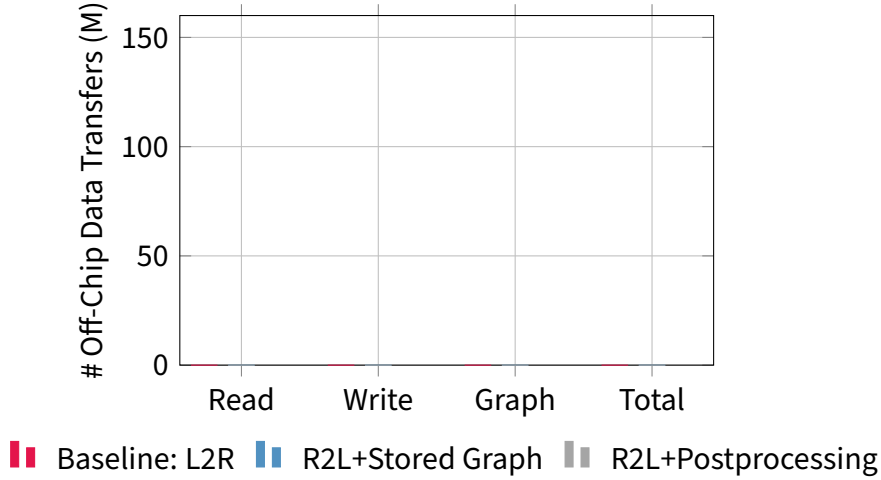




☑ Security Proof in Paper

Off-Chip Data Transfers



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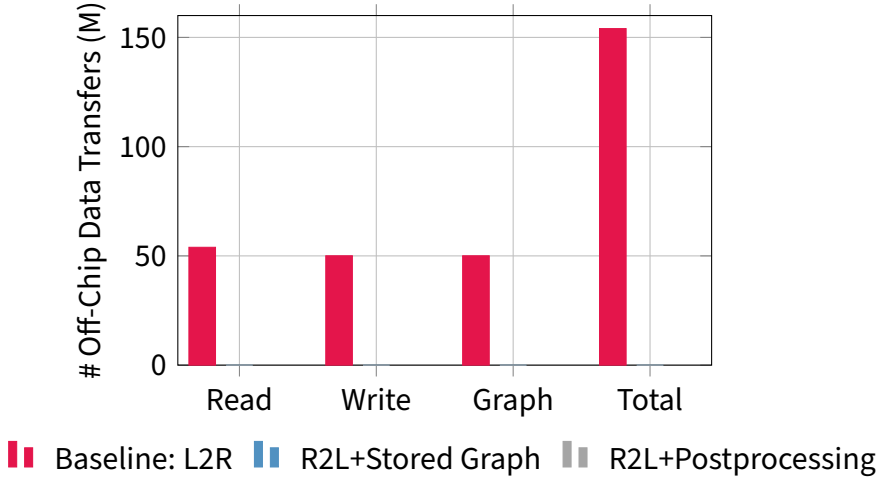

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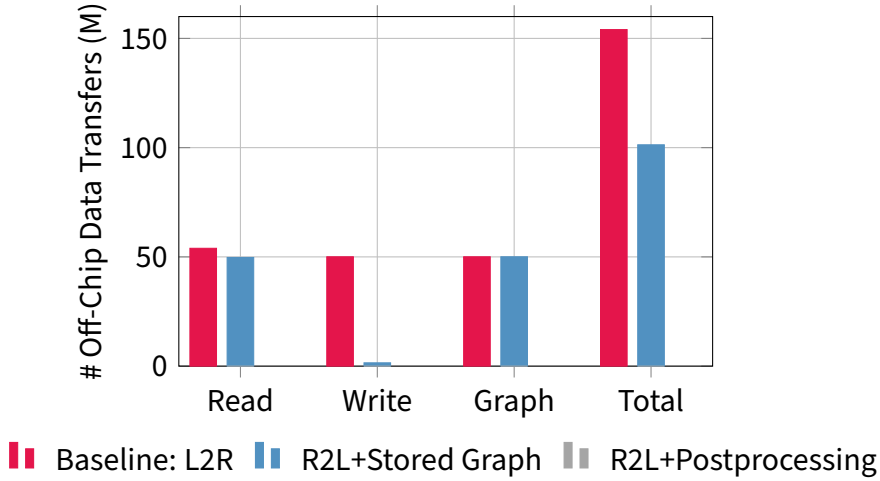

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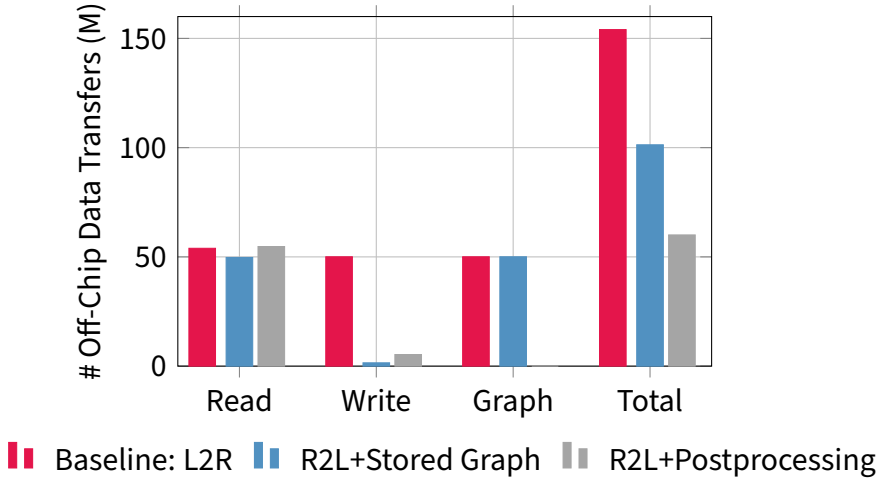

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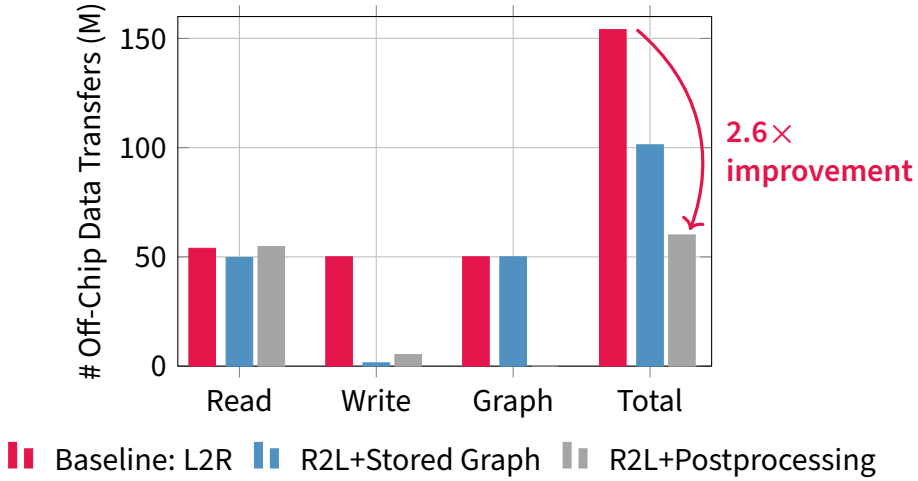
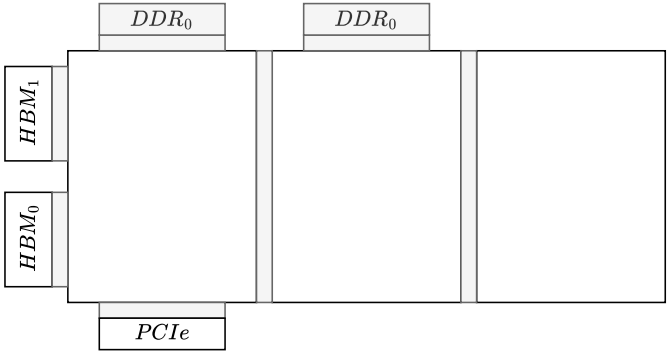


Hardware Architecture

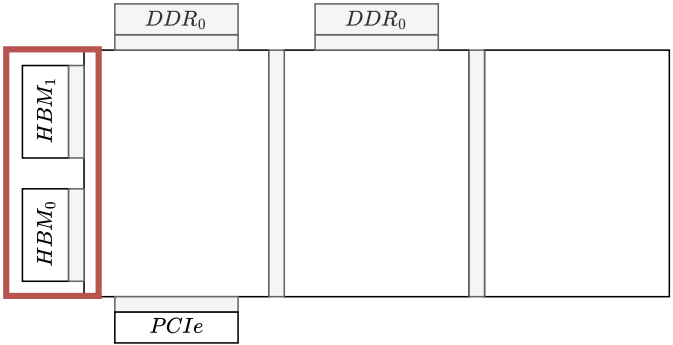
Hardware Architecture and Unit Placement



Alveo U280 FPGA

- HBM2 (460 GB/s)
- DDR4 (2x19 GB/s)
- 3 x SLR-Regions
- 20K SLL Lines à SLR

Hardware Architecture and Unit Placement



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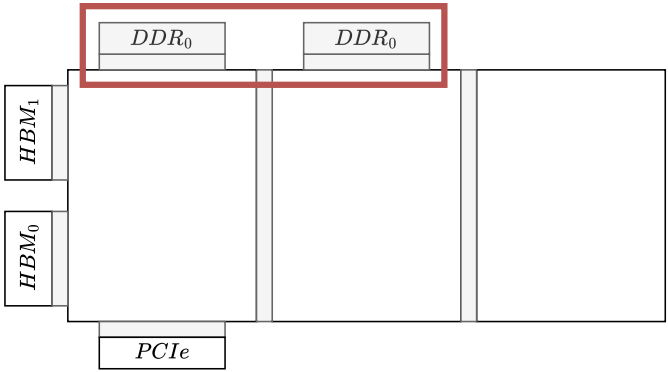

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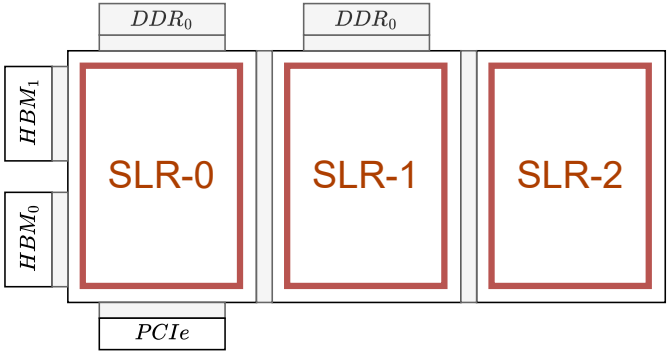

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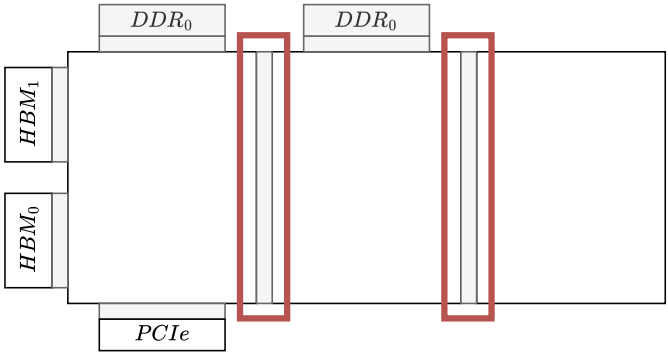

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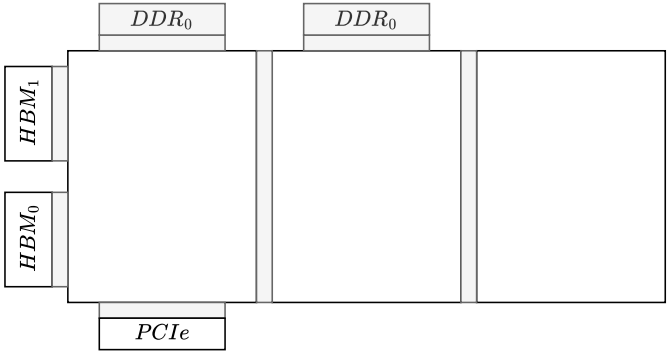

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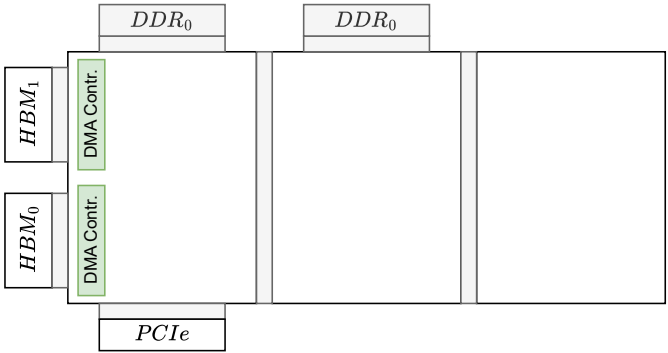

Alveo U280 FPGA

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→ Requires synchronization of 32 AXI-MM and 32 AXI-ST interfaces
→ Data movement of 16K bits per cycle @ 200 MHz

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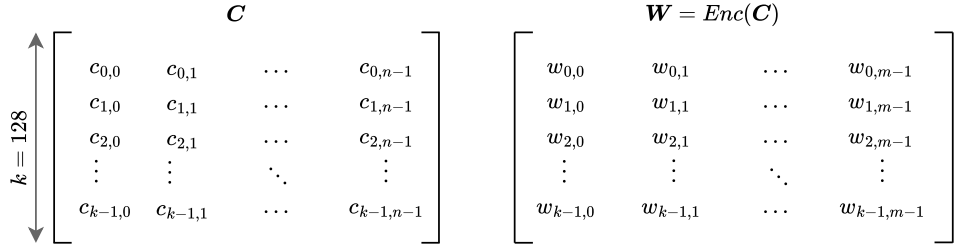

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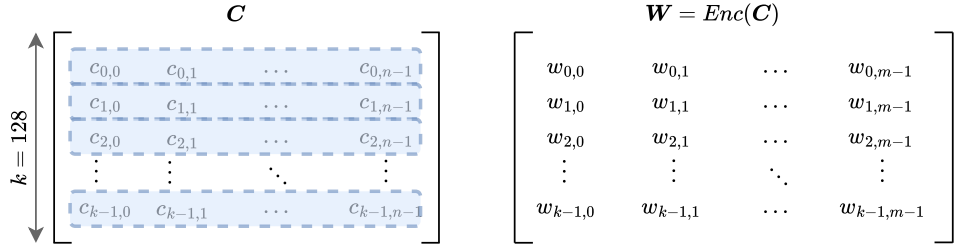

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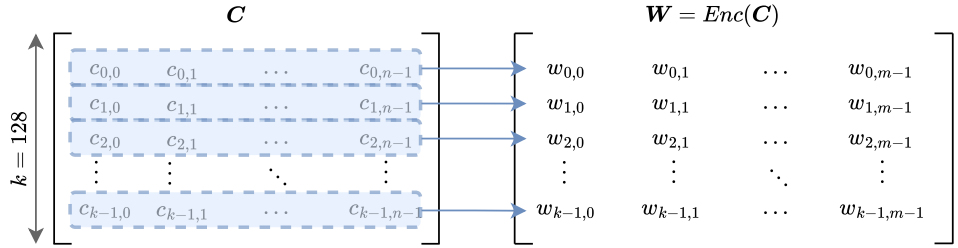

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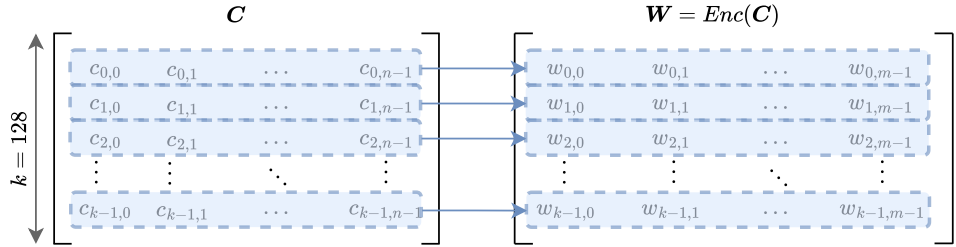

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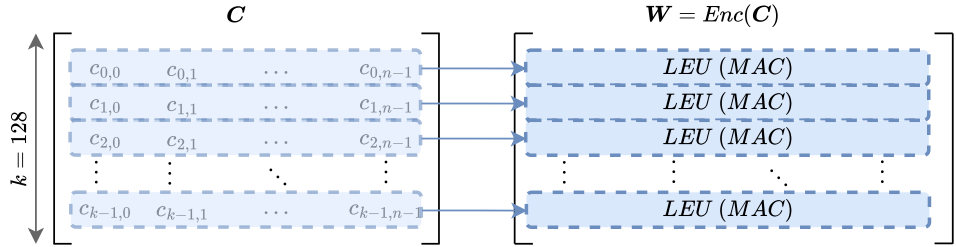
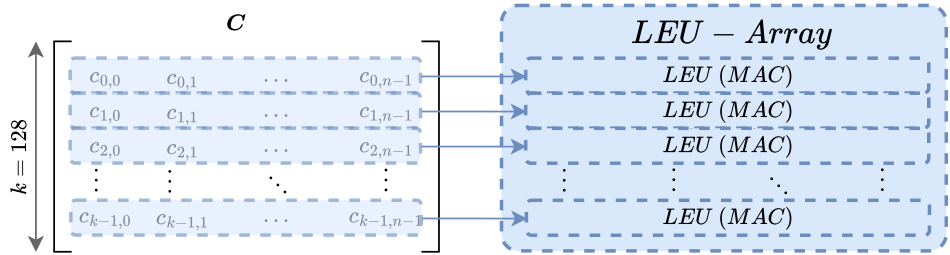


Hardware Architecture and Unit Placement



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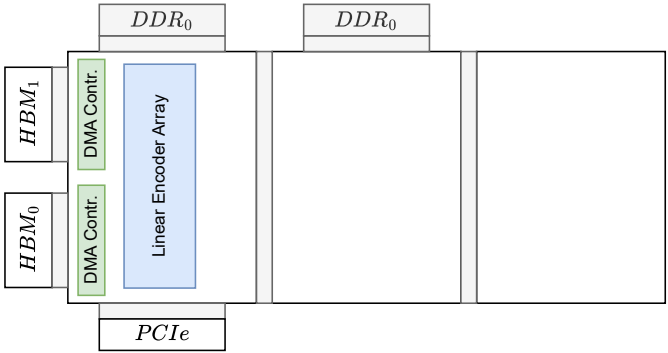
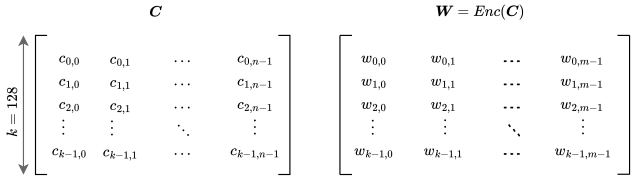


Alveo U280 FPGA

- HBM2 (460 GB/s)
- DDR4 (2x19 GB/s)
- 3 x SLR-Regions
- 20K SLL Lines à SLR

→ Requires 32K IO lines (16K input and 16K output)

Hardware Architecture and Unit Placement



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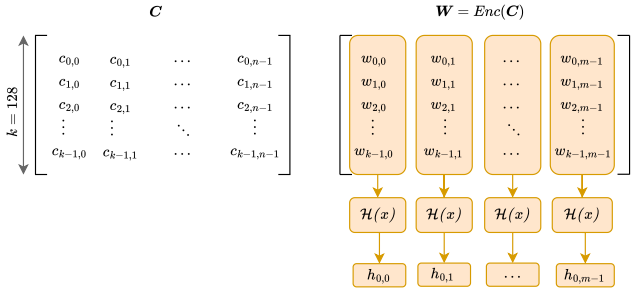

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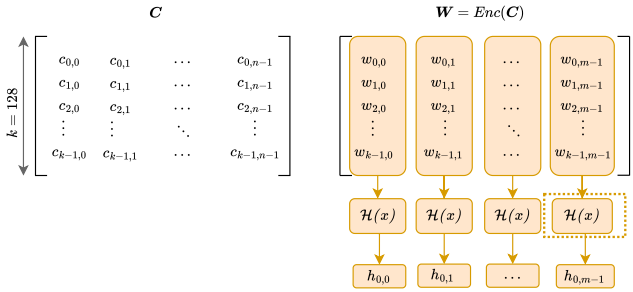

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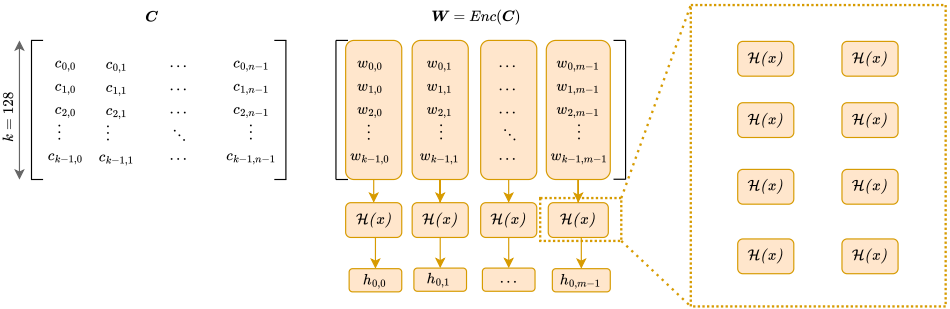

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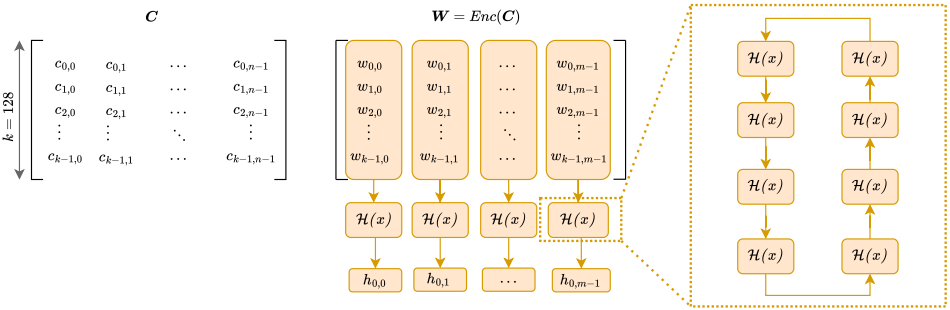

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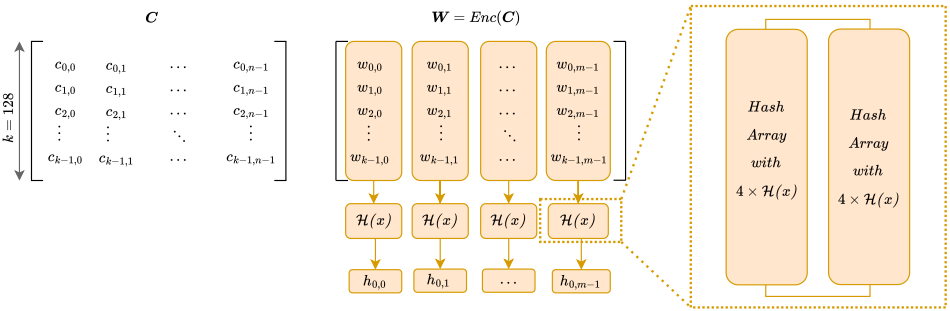

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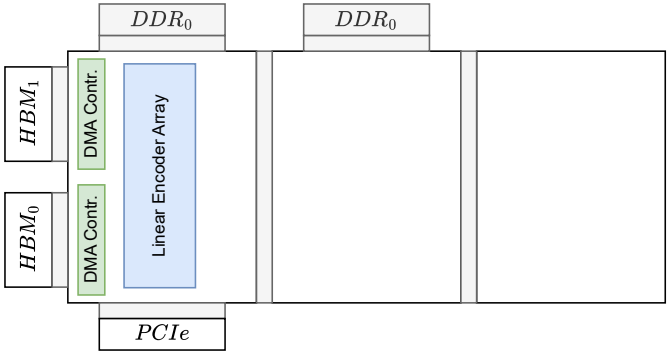

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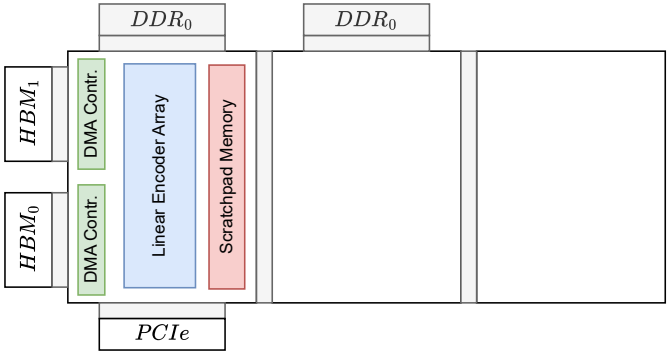

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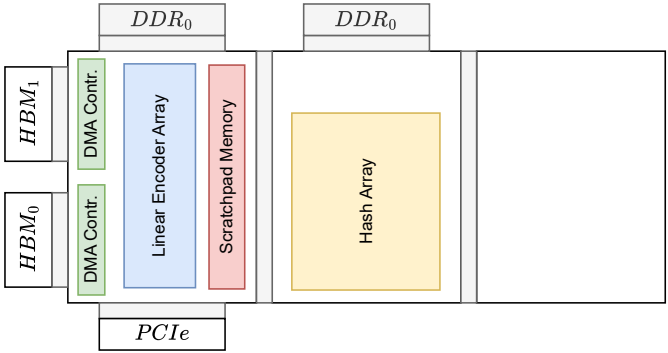
→ Scratchpad reshapes data from 512x32 to 1088x16 (muxed to 1088x8)

Hardware Architecture and Unit Placement



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→ Hash Array requires a total of 12K (1088x8 + 1600x2)

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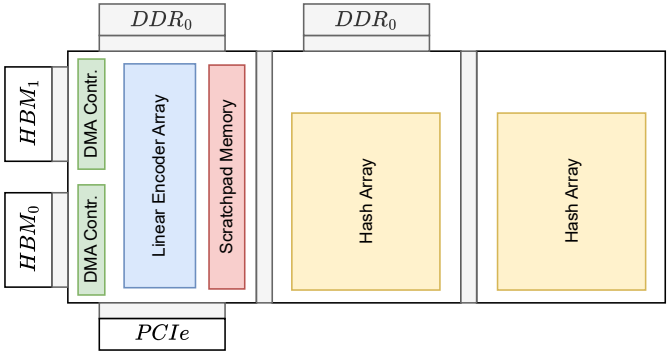

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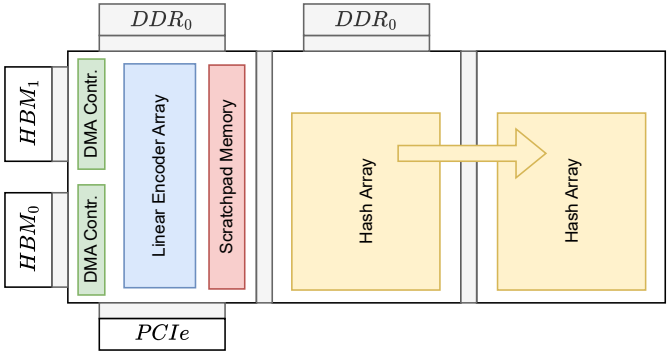

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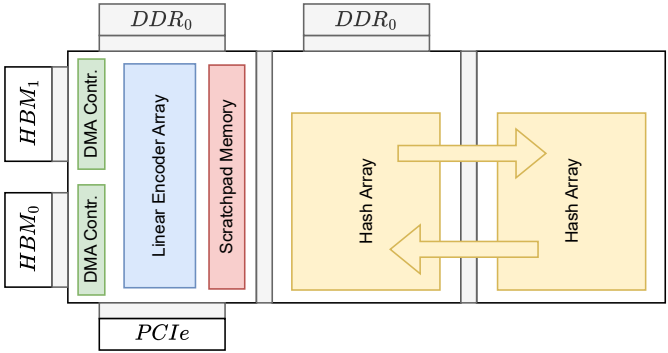

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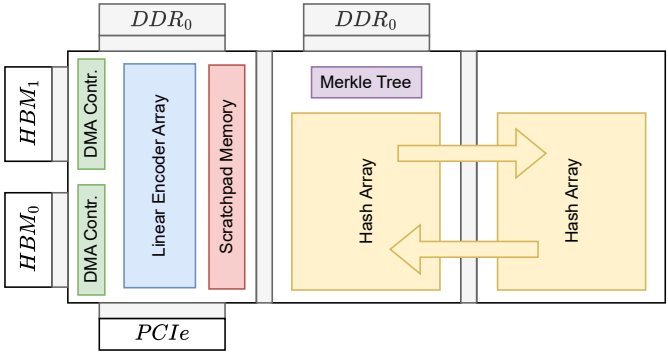

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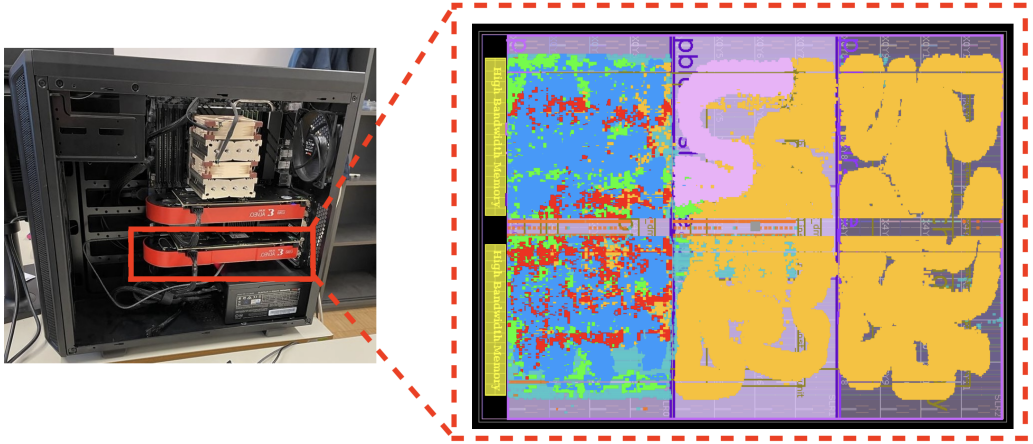

Speedup Results over Software

- **Commitment: up to 264×**

  Linear Encode: up to 231×
  Leaf Hash: up to 1,015×
  Merkle Tree: up to 21×

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  Linear Encode: up to 231×
  Leaf Hash: up to 1,015×
  Merkle Tree: up to 21×

- **Proving: up to 65×**

Conclusion

- Multi-SLR architecture with HBM memory
- First FPGA accelerator for PCS with linear prover time
- Inverted expander graphs + Proof
- Reducing off-chip memory accesses

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SCIENCE
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Accelerating Hash-Based Polynomial Commitment Schemes with Linear Prover Time

Florian Hirner **Florian Krieger** Constantin Piber Sujoy Sinha Roy

Graz University of Technology

CHES 2025 Conference

› isec.tugraz.at

Demo

Bonus Slides

Spielman Encoding